Wave Propagation Across Velocity Gradients in Neutron Star Magnetospheres

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Abstract

The presence of orthogonal polarisations in observations of pulsar radio emission is commonly associated with an incoherent superposition of the natural wave modes of the source plasma. The inhomogeneous nature of this plasma is invoked firstly to effect the coupling of energy from one mode, initially dominant during wave growth, into two modes, and, secondly, to separate the paths of the two modes as they propagate out of the magnetosphere. Gradients perpendicular to the local magnetic field direction in the streaming velocity of the $e^+ - e^-$ plasma are explored as a mechanism for these effects.

1. Introduction

IN the linear theory of wave propagation in a medium, waves may grow due to the presence of complex solutions to the dispersion equation or dissipation of kinetic energy through resonant interactions with the plasma particles. The wave amplitude grows as $\exp(\Gamma s)$, where s is the distance along the propagation direction and there is a growth rate Γ associated with each mode. In practice the mode with the highest growth rate will come to dominate the emission after a few growth lengths $1/\Gamma$. Thus, in the case of pulsars where the radio emission is thought to originate through some instability in the magnetised plasma surrounding a rotating neutron star, a single dominant mode is expected to determine the polarisation of the resultant radiation. Observations show that this is clearly not the case and so a mechanism is required to transfer energy between the modes.

In an inhomogeneous plasma the polarisation properties of the natural modes may vary along the propagation path, and if this occurs in a distance $< O(1/\Delta k)$, where Δk is the difference in propagation constants for the two modes, then they are in some sense poorly defined and they no longer propagate independently. Waves in a particular mode may also encounter regions where they become evanescent and will be reflected by such layers. As this occurs under different conditions for different modes it can seperate their paths of propagation significantly.

The production of $e^+ - e^-$ pairs in neutron star magnetospheres is thought to occur through decay of γ-rays on the super-strong magnetic field near the stellar surface. Particles are essentially restricted to motion along field lines and so there is little interaction in the direction transverse to this. Thus, the nature of the plasma produced is expected to vary significantly across the open field line region due to variations in the electrodynamics of the pair-production process, and gradients in the bulk plasma velocity are one example of this.

2. Governing Equations

- This treatment of wave propagation follows closely that used in the study of radio waves in the ionosphere, with the main difference being the inclusion of the streaming motion of the particles and the presence of both electrons and positrons.
- The $e^+ - e^-$ plasma is modelled as a cold, magnetised, two component fluid, and is assumed stratified in the z direction with the magnetic field lying in the x direction. The governing equations of the problem are an equation of motion for each species,

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_\pm \cdot \nabla\right)(\gamma \mathbf{v}_\pm) = \pm\frac{e}{m}(\mathbf{E} + \mathbf{v}_\pm \times \mathbf{B}), \qquad (1)$$

a corresponding continuity equation,

$$\frac{\partial n_\pm}{\partial t} + \nabla \cdot (n_\pm \mathbf{v}_\pm) = 0, \qquad (2)$$

and the Maxwell equations,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad (3)$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_0 e \sum_{s=\pm} s n_s \mathbf{v}_s, \qquad (4)$$

where $\mathbf{v}$ is the particle velocity and n is the particle number density.
- After Fourier transforming in time and the x and y coordinates, the linearised set of the original governing equations reduces to four equations for the four non-redundant variables, which are the electric and magnetic field components E_x, E_y, B_x, and B_y. Defining a vector $\mathbf{e} = (E_y, cB_x, E_x, cB_y)^T$, where c is the speed of light, the problem may be cast in matrix form

$$\mathbf{e}' = \mathbf{T}\mathbf{e}, \qquad (5)$$

where the prime denotes differentiation w.r.t. the variable $\zeta = (\omega/c)z$, ω being the wave frequency. The non-zero components of the 4×4 matrix $\mathbf{T}$ are

$$\begin{aligned}
T_{12} &= -i,\\
T_{21} &= i\left[A^2 - 1 + \frac{BX}{\gamma(B^2 - Y^2)}\frac{B(B^2-Y^2) - X(\eta X - Y^2)}{B^2 - Y^2 - \eta XB}\right],\\
T_{24} &= -\frac{XY(\beta - A)}{B^2 - Y^2 - \eta XB},\\
T_{31} &= \frac{ABXY}{\gamma(B^2 - Y^2 - \eta XB)},\\
T_{34} &= -i\left[A^2 - 1 - \frac{\eta ABX(\beta - A)}{B^2 - Y^2 - \eta XB}\right],\\
T_{42} &= -\frac{XY\beta}{B^2 - Y^2 - \eta\frac{X}{\gamma}},\\
T_{43} &= i\left[1 - \frac{X}{\gamma B^2}\left(1 - \frac{\eta\gamma\beta(\beta - A)(X - \gamma B^2)}{B^2 - Y^2 - \eta\frac{X}{\gamma}}\right)\right].
\end{aligned} \qquad (6)$$

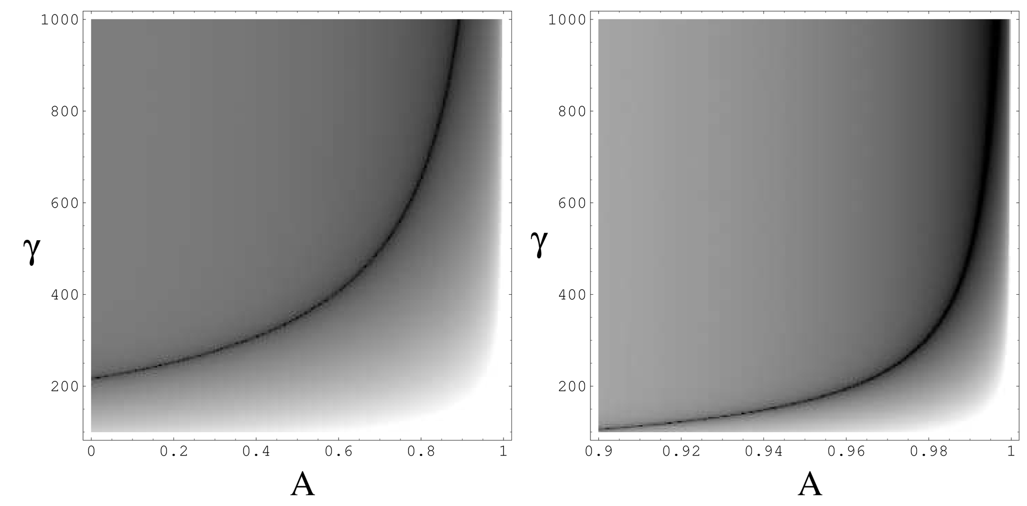
- With ω_p being the plasma frequency, Ω_e the electron cyclotron frequency, and γ and β being the Lorentz factor and velocity respectively associated with the streaming motion of the plasma, we have the definitions $A = k_x c/\omega$, $B = \gamma(1 - A\beta)$, $X = \omega_p^2/\omega^2$, and $Y = \Omega_e/\omega$. η is the average charge of the particles.
- This form is valid so long as the length scale over which the plasma parameters vary is much longer than the vacuum wavelength, so that derivatives in these quantities may be neglected.
- To determinie the parameters in the problem the propagation region is chosen to lie on the axis of a dipole magnetic field aligned with the rotation axis of the neutron star. The background particle number density is taken as the Goldreich-Julian number density multiplied by a multiplicity factor $M = 1/\eta$. The neutron star has a rotational period of 1s and a magnetic field strength of 10^8T at the stellar surface (radius $r = R$).
- Wave propagation in the strong field regions of neutron star magnetospheres is often investigated in the limit in which $B \to \infty$, however the inclusion of a finite magnetic field strength is essential to achieve mode coupling.

3. Dispersion Characteristics

- The eigenvalues q of the matrix $\mathbf{T}$ correspond to the modes of a locally homogeneous medium and are given by

$$\begin{aligned}
q^2 = &\frac{1}{2}(T_{12}T_{21} + T_{24}T_{42} + T_{34}T_{43})\\
&\pm\frac{1}{2}\Big[(T_{12}T_{21} + T_{24}T_{42} + T_{34}T_{43})^2\\
&-4T_{12}T_{43}(T_{21}T_{34} - T_{24}T_{31})\Big]^{1/2}.
\end{aligned} \qquad (7)$$

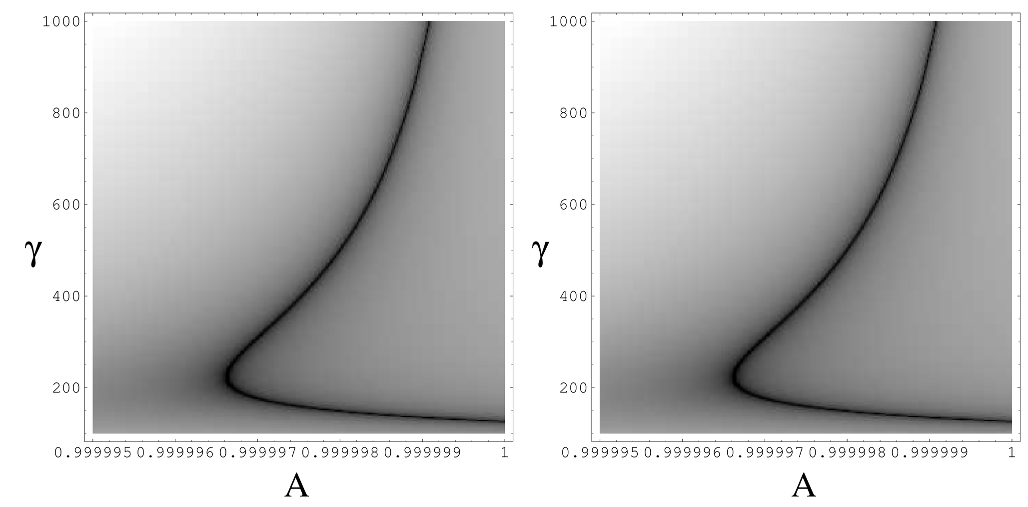
- Solutions are in general complex, and correspond to pairs of the x- and o-modes which propagate/decay in opposite directions. A particular mode will propagate only where the associated eigenvalue is pure imaginary.
- Of particular interest are points where the eigenvalues of two of the modes become degenerate, as strong coupling will occur between them in the surrounding regions. When the term in the square brackets in (7) is zero the codirectional x- and o-mode solutions become pairwise degenerate (see Figure 1).



(a) (b)

Figure 1: *Grey-scale plots of the Log of the absolute value of the term in square brackets in (7). The black region denotes where this term $\to 0$, i.e. where strong coupling occurs. Above this the eigenvalues are complex (both real and imaginary parts) and below they are real or pure imaginary. $r = 10R$ in (a) and $r = 100R$ in (b). $\eta = 10^{-3}$ in both.*

- Inspection of (6) and (7) implies that satisfaction of the condition $\gamma^3(1 - A\beta)^2 = X$ leads to the degeneracy of the forward and backward propagating o-mode solutions (see Figure 2). Strong reflections will thus occur even if the waves do not reach a layer where they are evanescent.



(a) (b)

Figure 2: *Grey-scale plots of the Log of the absolute value of $\gamma^3(1 - A\beta)^2 - X$. The black region denotes where this term $\to 0$ and thus where the o-mode is strongly reflected. To the left of this the eigenvalues are pure imaginary and to the right they are real. $r = 10R$ and $\eta = 1$ in (a), and $r = 100R$ and $\eta = 10^{-3}$ in (b).*

4. Reflections Off Sharp Boundaries

- If the transition between two homogeneous regions of plasma occurs over a distance $\ll \omega/c$ then one may simply enforce continuity of the transverse (to z) field components across the sharp boundary.
- To demonstrate the strong reflection of the o-mode at near-parallel propagation to the magnetic field direction one may calculate transmission and reflection coefficients by normalising the eigenvectors to a given value of the z-component of the time-averaged Poynting vector.

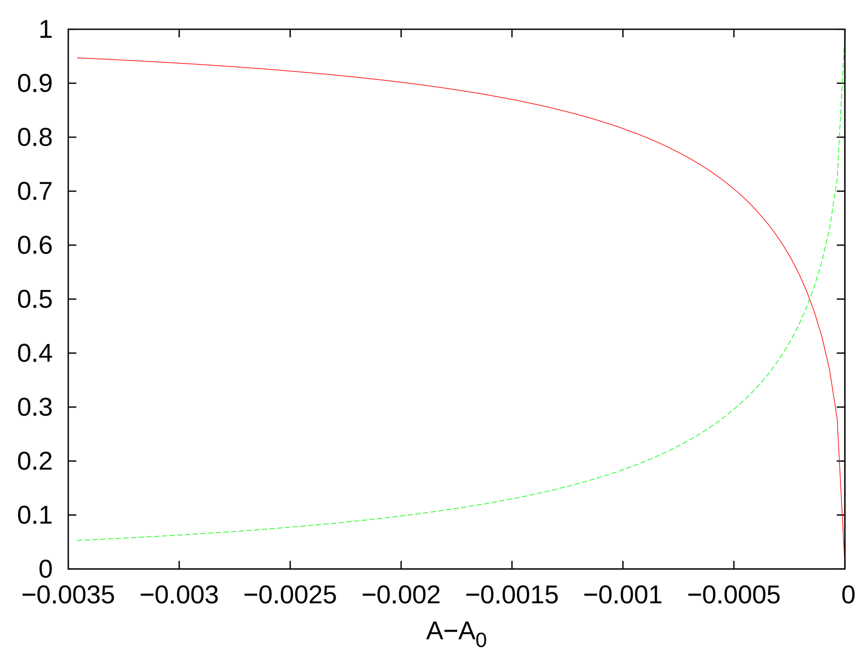


Figure 3: *Reflection (green) and transmission (red) coefficients for the o-mode incident upon a shear in bulk plasma velocity. The region of the incident wave has $\gamma = 600$ which bounds plasma with $\gamma = 800$. $r = 100R$ and $\eta = 10^{-3}$. The eigenvalues are imaginary in both regions. A_0 is the degeneracy point.*

- If an o-mode wave is propagating sufficiently close to the magnetic field it requires only a small variation in the Lorentz factor of the streaming plasma to confine it to a narrow slab. The o-mode will then be 'ducted' along the magnetic field lines, while the x-mode propagates unperturbed.

5. Conclusions

- Variations in bulk plasma velocity perpendicular to the magnetic field direction can strongly affect the polarisation properties of the natural wave modes.
- Coupling between the x- and o-modes can occur at and around points where they become degenerate.
- Ducting of the o-mode along magnetic field lines is possible due to strong reflections off boundaries between plasma regions with different streaming velocities at near parallel propagation.

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