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Electromagnetic Fields of Magnetized Compact Stars in Braneworld

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Plan of talk

- Basic assumptions and approach
- Electromagnetic Fields of Magnetized Star in Brane
- Stationary Solutions to Maxwell Equations for Magnetized Highly Conducting Spherical Star in Braneworld
- Astrophysical Consequences
- Conclusion

Assumptions and Approach

- GR effects are important for compact objects

$$R \sim 10\,km\,,\ R_g = \frac{2GM}{c^2 r} \sim 5\,km\,,\ (1-\frac{2GM}{c^2 r}) \sim 0.5$$

- Immense difficulty of simultaneously solving the Maxwell eqs and the highly nonlinear Einstein eqs
- Perfect conductivity in the interior region of the star
- The exterior is considered to be as vacuum
- Gravitational field feedback is neglected

- # Braneworld

Recent progress of superstring theory says that our four dimensional universe is entirely restricted to a brane inside a higher dimensional space, called the bulk. According to RS model, the universe is 5D AdS space and elementary particles except of graviton is localized at 3+1 spacetime.

The new picture of the universe

METRIC FOR SPHERICAL RELATIVISTIC STAR IN BRANEWORLD

$$ds^2 = -A^2(r)dt^2 + H^2(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

EINSTEIN EQS FOR UNKNOWN METRIC FUNCTIONS A (r) & H(r)

$$\frac{1}{r^2} - \frac{1}{H^2}\left(\frac{1}{r^2} - \frac{2}{r}\frac{H'}{H}\right) = 8\pi\rho^{eff} \ ,$$

$$-\frac{1}{r^2} + \frac{1}{H^2}\left(\frac{1}{r^2} + \frac{2}{r}\frac{A'}{A}\right) = 8\pi\left(\rho^{eff} + \frac{4}{k^2\lambda}P\right) \ ,$$

$$p' + \frac{A'}{A}(\rho + p) = 0 \ ,$$

$$U' + 4\frac{A'}{A}U + 2P' + 2\frac{A'}{A}P + \frac{6}{r}P = -2(4\pi)^2(\rho + p)\rho' \ .$$

nonlocal energy density U and nonlocal pressure P.

ρ^{eff} is the effective total energy density,

ρ and p are matter energy density and pressure

OUTSIDE THE STAR

$$A^2(r) \equiv \left(1 - \frac{2M}{r} + \frac{Q}{r^2}\right) = H^{-2}(r)\ , \qquad r > R\ ,$$

negative Weyl "charge" $Q = -3MR\rho/\lambda$

M and R are the total mass

and radius of the star, λ is the brane tension.

HORIZON

$$r_+ = M\left(1+\sqrt{1-\frac{Q}{M^2}}\right)$$

MAXWELL EQS

$$e^{\alpha\beta\mu\nu} F_{\beta\mu,\nu} = 0,$$

$$F^{\alpha\beta}{}_{;\beta} = 4\pi J^{\alpha}$$

FIRST PAIR

$$\sin\theta\left(r^2 B^{\hat{r}}\right)_{,r} + Hr\left(\sin\theta B^{\hat{\theta}}\right)_{,\theta} + HrB^{\hat{\phi}}{}_{,\phi} = 0\ ,$$

$$(r\sin\theta)\,\frac{\partial B^{\hat{r}}}{\partial t} = A\left[E^{\hat{\theta}}{}_{,\phi} - \left(\sin\theta E^{\hat{\phi}}\right)_{,\theta}\right]\ ,$$

$$(Hr\sin\theta)\,\frac{\partial B^{\hat{\theta}}}{\partial t} = -AHE^{\hat{r}}{}_{,\phi} + \sin\theta\left(rAE^{\hat{\phi}}\right)_{,r}\ ,$$

$$(Hr)\,\frac{\partial B^{\hat{\phi}}}{\partial t} = -\left(rAE^{\hat{\theta}}\right)_{,r} + AHE^{\hat{r}}{}_{,\theta}$$

SECOND PAIR

$$\sin\theta\left(r^2 E^{\hat{r}}\right)_{,r} + Hr\left(\sin\theta E^{\hat{\theta}}\right)_{,\theta} + HrE^{\hat{\phi}}{}_{,\phi} = 4\pi Hr^2\sin\theta J^{\hat{t}} \ ,$$

$$A\left[\left(\sin\theta B^{\hat{\phi}}\right)_{,\theta} - B^{\hat{\theta}}{}_{,\phi}\right] = (r\sin\theta)\frac{\partial E^{\hat{r}}}{\partial t} + 4\pi Ar\sin\theta J^{\hat{r}} \ ,$$

$$AHB^{\hat{r}}{}_{,\phi} - \sin\theta\left(r\ AB^{\hat{\phi}}\right)_{,r} = (Hr\sin\theta)\frac{\partial E^{\hat{\theta}}}{\partial t} + 4\pi AHr\sin\theta J^{\hat{\theta}} \ ,$$

$$\left(ArB^{\hat{\theta}}\right)_{,r} - AHB^{\hat{r}}{}_{,\theta} = (Hr)\frac{\partial E^{\hat{\phi}}}{\partial t} + 4\pi AHrJ^{\hat{\phi}} \ .$$

Dipolar Magnetic Field and Separable Solutions

$$B^{\hat{r}}(r,\theta) = F(r)\cos\theta,$$

$$B^{\hat{\theta}}(r,\theta) = G(r)\sin\theta,$$

$$B^{\hat{\phi}}(r,\theta) = 0.$$

MAXWELL EQS FOR RADIAL FUNCTIONS

$$\left(r^2 F\right)_{,r} + 2HrG = 0 \ ,$$

$$\left(rAG\right)_{,r} + AHF = 0 \ .$$

Interior Analytical Solution

$$F = \frac{C_1}{R^3}\mu \, , \qquad G = -\frac{H^{-1}C_1}{R^3}\mu = -H^{-1}F$$

For Stiff Matter

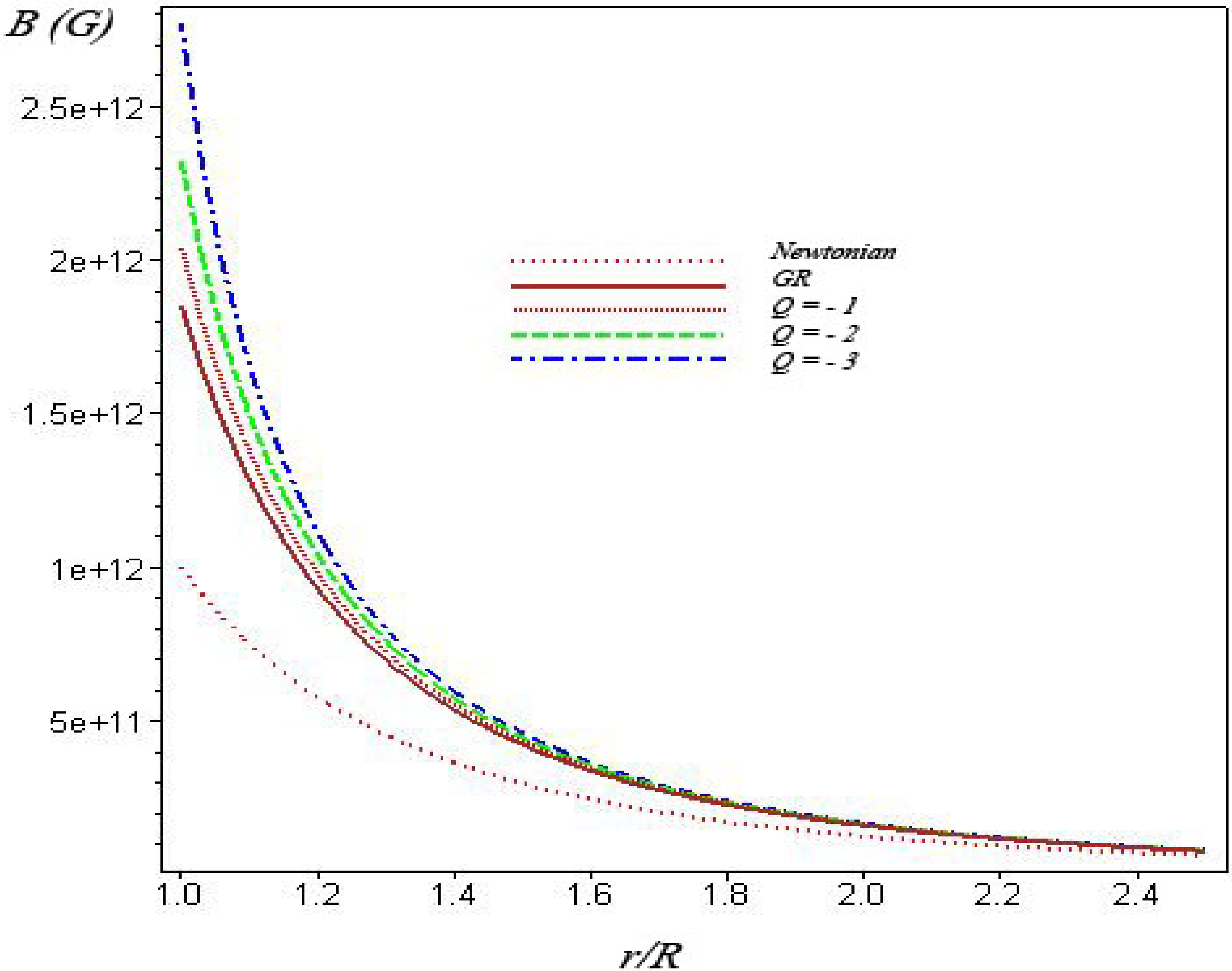
$$\rho^{eff} = p^{eff} + \frac{4}{k^4\lambda}P$$

Exterior Numerical Solution for Magnetic Field

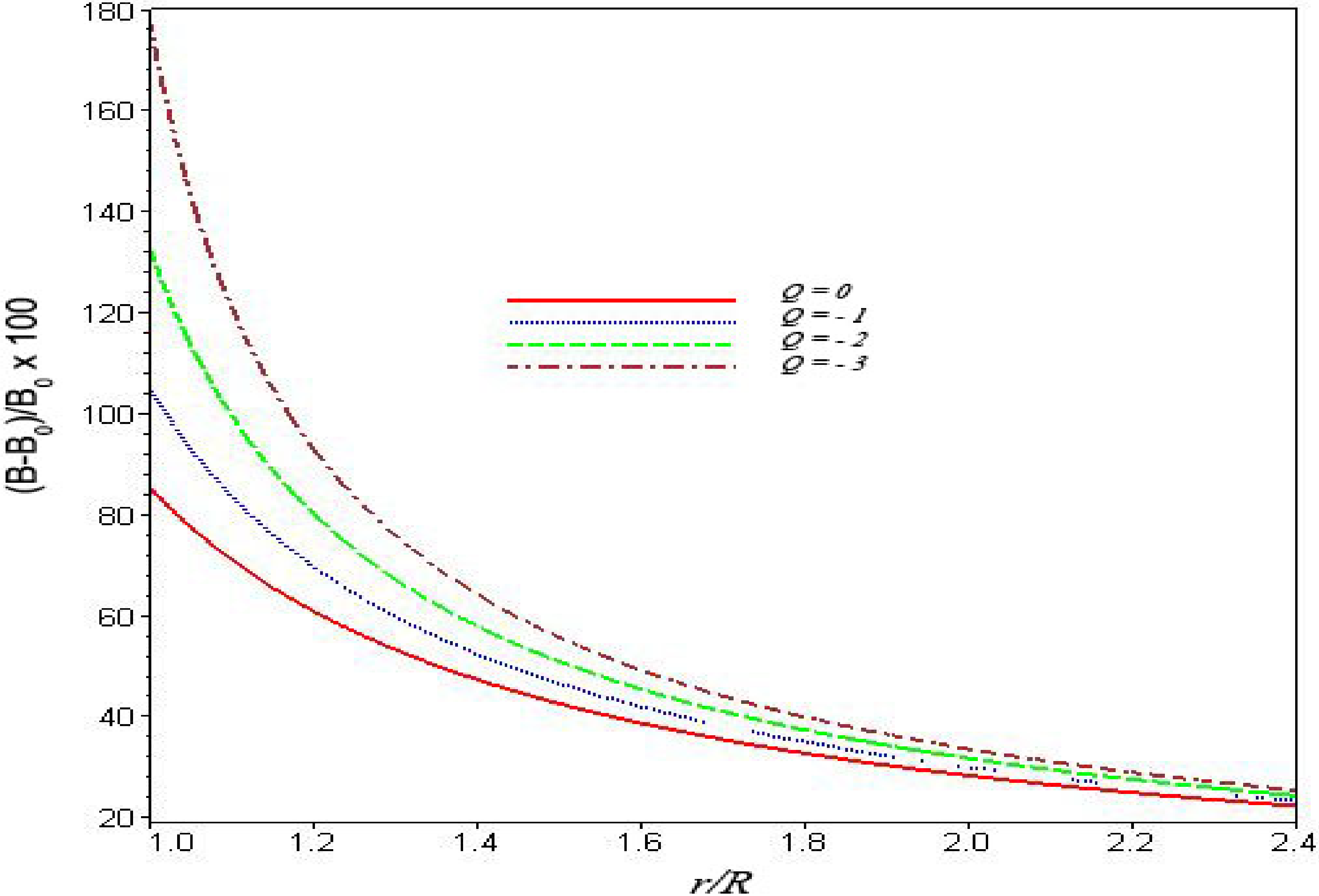
$$\frac{d}{dr}\left[\left(1-\frac{2M}{r}+\frac{Q}{r^2}\right)\frac{d}{dr}\left(r^2F\right)\right]-2F=0$$

The analytical solution exists, when Q=0:

$$\frac{F_{GR}(r)}{F_{Newt}(r)}=-\frac{3R^3}{8M^3}\left[\ln N^2+\frac{2M}{r}\left(1+\frac{M}{r}\right)\right]$$



B (G)
2.5e+12
2e+12
1.5e+12
1e+12
5e+11
Newtonian
GR
Q = - 1
Q = - 2
Q = - 3
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
r/R



(B-B0)/B0 x 100
r/R
Q = 0
Q = - 1
Q = - 2
Q = - 3
180
160
140
120
100
80
60
40
20
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4

Interior Numerical Solution

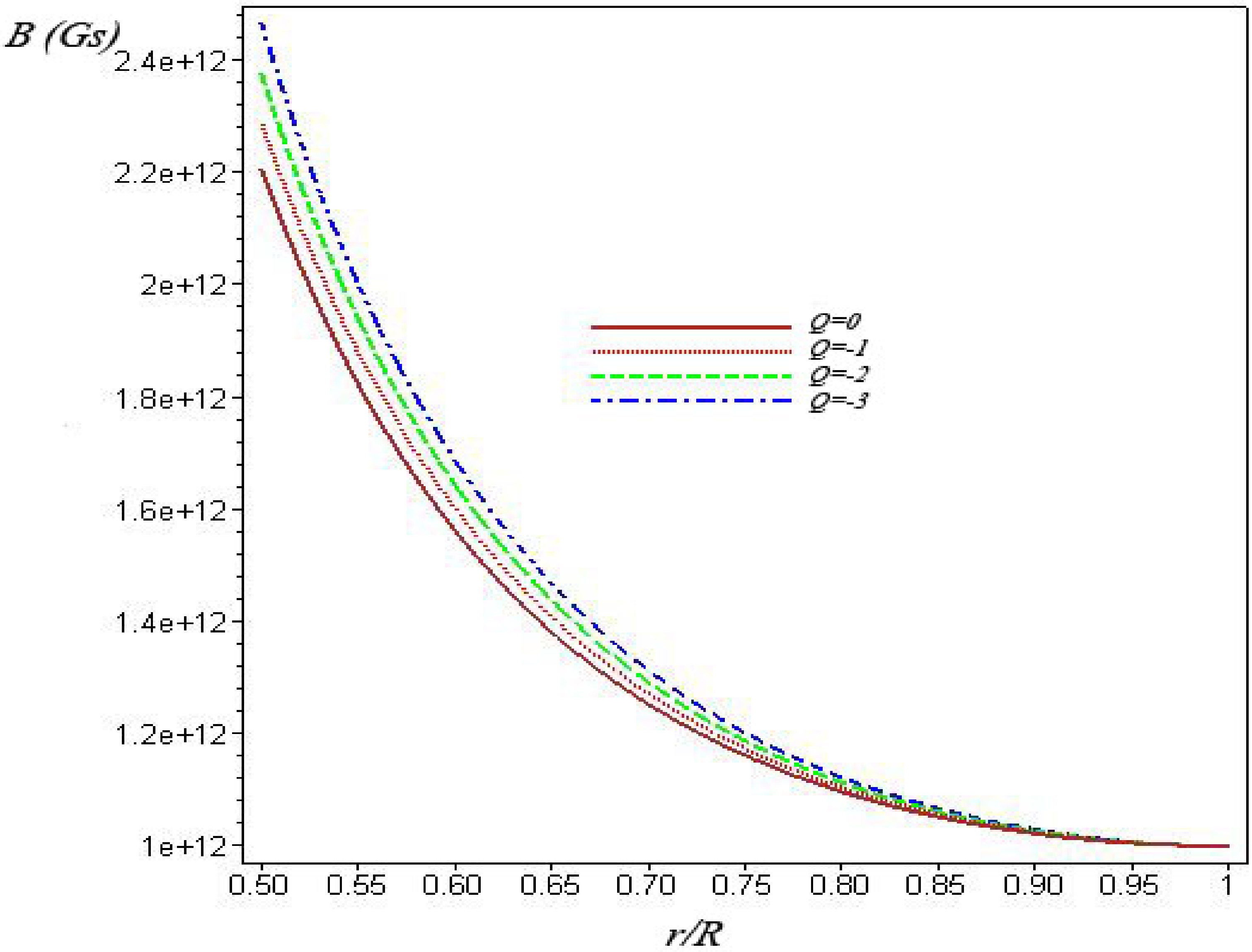
$$\frac{d}{dr}\left[\frac{\Delta(r)}{\rho + p(r)}\frac{d}{dr}\left(r^2 F\right)\right] - \frac{2F}{\left[\rho + p(r)\right]\Delta(r)} = 0$$

where

$$\Delta(r) \equiv \left[1 - \frac{2M}{r}\left(\frac{r}{R}\right)^3\left[1 - \frac{Q}{6MR}\right]\right]^{1/2},$$

$$p(r) = \frac{\left[\Delta(r) - \Delta(R)\right]\left(1 - Q/3MR\right)\rho}{\left[3\Delta(R) - \Delta(r)\right] - \left[3\Delta(R) - 2\Delta(r)\right]Q/3MR},$$

$$m(r) = M\left[1 - \frac{Q}{6MR}\right]\left(\frac{r}{R}\right)^3.$$



B (Gs)
2.4e+12
2.2e+12
2e+12
1.8e+12
1.6e+12
1.4e+12
1.2e+12
1e+12
Q=0
Q=-1
Q=-2
Q=-3
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1
r/R

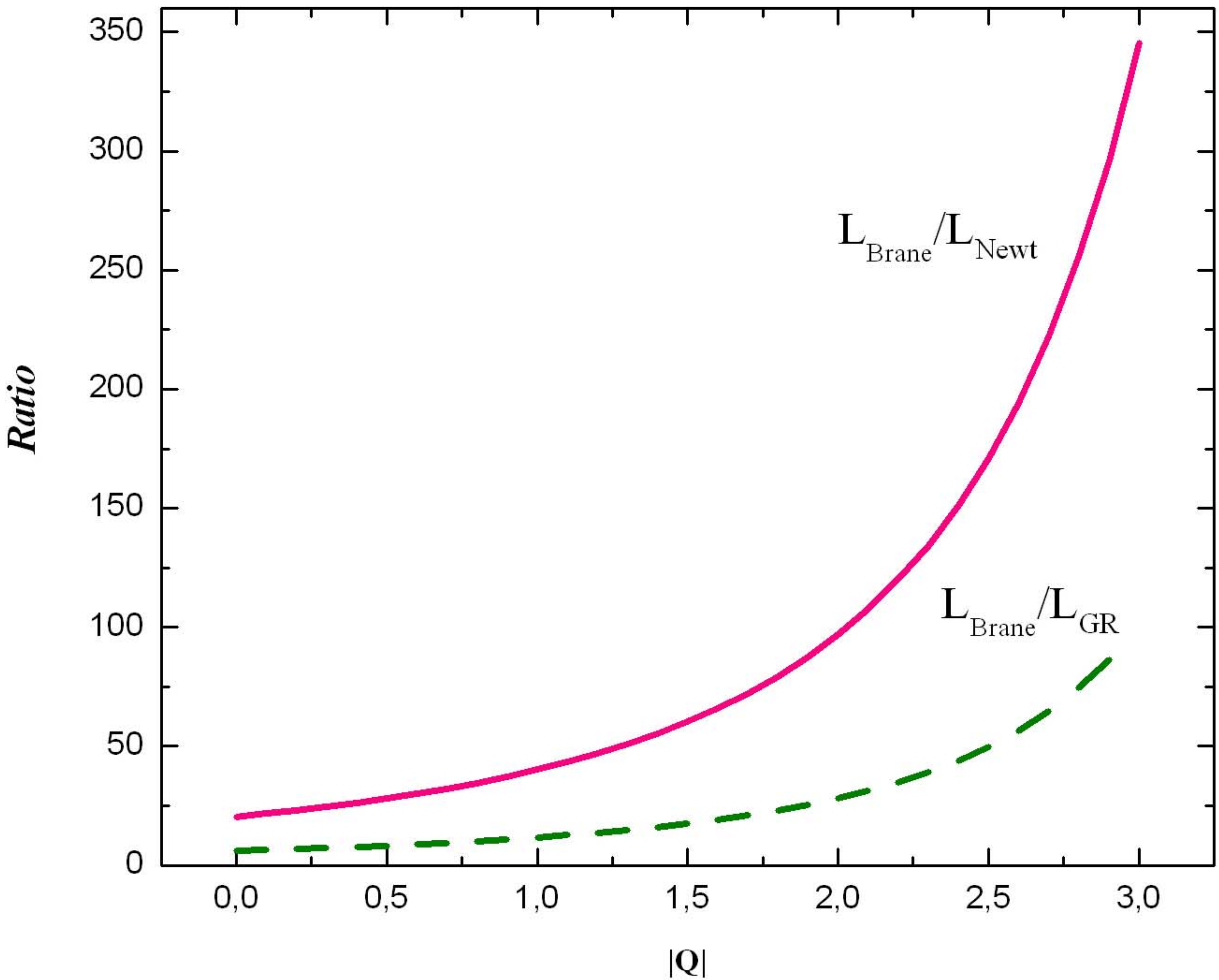
Astrophysical Consequences

Luminosity of the relativistic star is

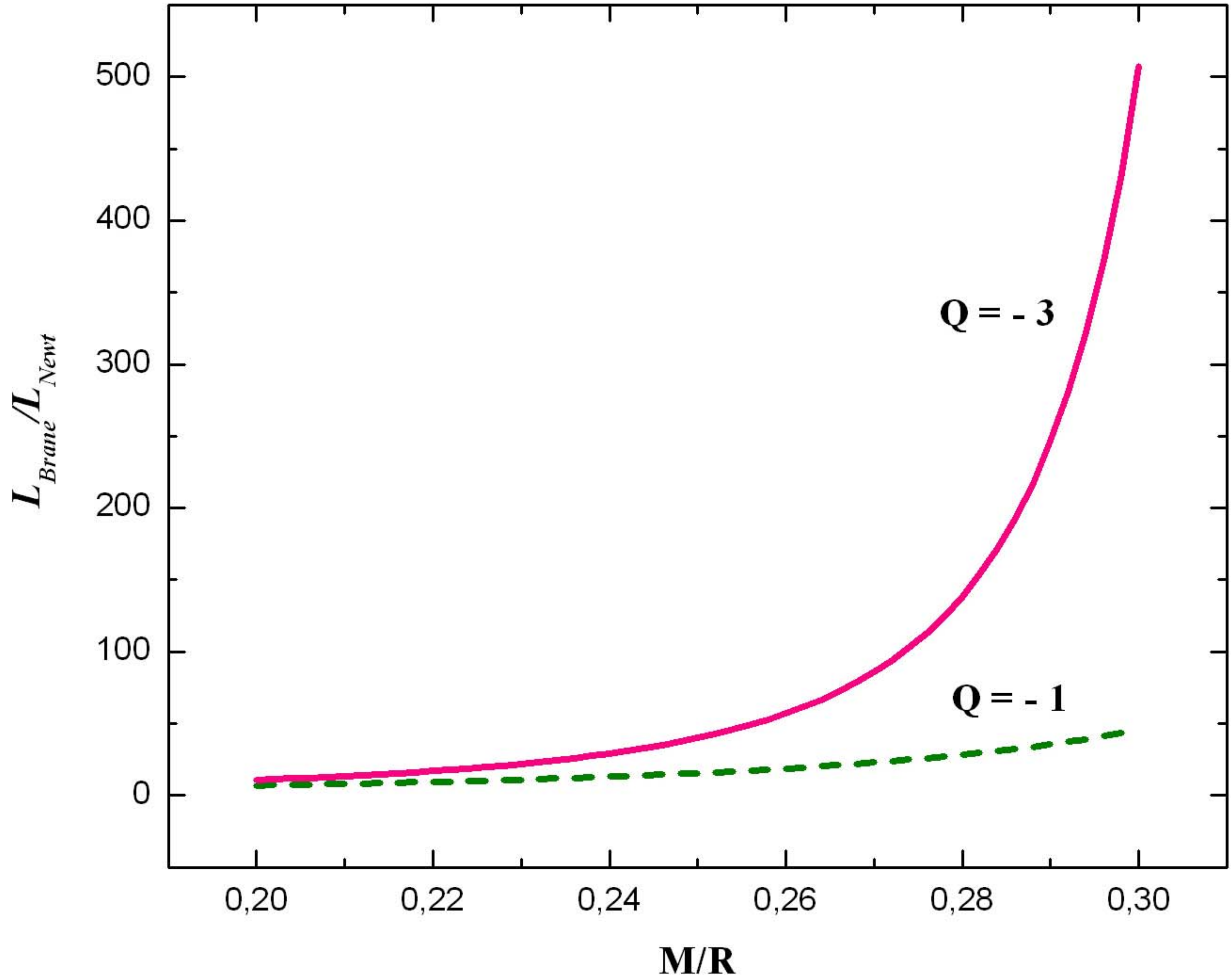
$$L_{em} = \frac{\Omega_R^4 R^6 \widetilde{B}_0^2}{6c^3} \sin^2 \chi$$

The rate of energy loss is

$$\frac{L_{em}}{\left(L_{em}\right)_{Newt}} = \left(\frac{F_R}{A_R^2}\right)^2$$



350
300
250
200
150
100
50
0
0,0
0,5
1,0
1,5
2,0
2,5
3,0
Ratio
|Q|
L_{Brane}/L_{Newt}
L_{Brane}/L_{GR}



L_{Brane}/L_{Newt}
500
400
300
200
100
0
0,20
0,22
0,24
0,26
0,28
0,30
M/R
Q = - 3
Q = - 1

CONCLUSION

- The effect of additional braneworld tension on electromagnetic fields is considered.
- The enhancement of magnetic field coming from the effect of brane tension depends on the parameter Q: For larger Q, the effect is stronger.
- There are two effects of brane tension on magneto-dipolar radiation: a) Due to the amplification of surface magnetic field by brane tension; b) Due to the presence of Q/r^2, in the red shift factor.
- The expression for magnetodipolar luminosity of rotating braneworld magnetized star gives enhancement up to two orders