Pulsar Braking Indices

A.M.Alpar,[a] and A.Baykal,[b]

[a]Faculty of Engineering and Natural Sciences, Sabanci University, Turkey

[b] Department of Physics, Middle east Technical University, Turkey

ABSTRACT

Almost all pulsars with anomalous positive $\ddot{\Omega}$ measurements (corresponding to anomalous braking indices in the range 5< n <100), including all the pulsars with observed large glitches ($\Delta\Omega/\Omega > 10^{-7}$) as well as post glitch or interglitch $\ddot{\Omega}$ measurements obey the scaling between $\ddot{\Omega}$ and glitch parameters originally noted in the Vela pulsar. Negative second derivative values can be understood in terms of glitches that were missed or remained unresolved. We discuss the glitch rates and a priori probabilities of positive and negative braking indices according to the model developed for the Vela pulsar. This behavior supports the universal occurrence of a nonlinear dynamical coupling between the neutron star crust and an interior superfluid component.

1 Introduction

Anomalous second derivatives of the rotation rates of radio pulsars may have interesting implications. Very large positive or negative second derivatives are likely to be artefacts of timing noise. We show here that second derivatives corresponding to braking indices n in the interval $5 < |n| < 100$ generally fit well with secular interglitch behaviour according to a model previously applied to the Vela pulsar. Pulsars with large glitches ($\Delta\Omega/\Omega \geq 10^{-7}$) and measured anomalous second derivatives of the rotation rate, mostly positive (Shemar & Lyne 1996, Lyne, Shemar & Graham-Smith 2000, Wang et al. 2000), as well as pulsars with positive or negative anomalous second derivatives but no observed glitches (Johnston & Galloway 1999) scale with the model. We infer that isolated neutron stars older than Vela have dynamical behaviour similar to the Vela pulsar. This implies relatively large energy dissipation rates that can supply a luminosity to older isolated neutron stars.

The spindown law of a pulsar is usually given in the form $\dot{\Omega} = -k\Omega^n$ where n, the braking index, is 3 if the pulsar spindown is determined purely by electromagnetic radiation torques generated by the rotating magnetic dipole moment of the neutron star. The braking index has been conventionally measured through the relation

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} \tag{1}$$

by measuring $\ddot{\Omega}$, the second derivative of the pulsar rotation frequency. An alternative method, suggested recently by Johnston & Galloway (1999) is based on integrating, rather than differentiating, the spindown law, to obtain

$$n = 1 + \frac{\Omega_1\dot{\Omega}_2 - \Omega_2\dot{\Omega}_1}{\dot{\Omega}_1\dot{\Omega}_2(t_2 - t_1)} \tag{2}$$

where Ω_i and $\dot{\Omega}_i$ are values measured at t_i.

Among the radio pulsars known, only young pulsars have braking indices measured with accuracy. These reported braking indices are all less than 3: For the Crab pulsar n= 2.509 ± 0.001 (Lyne, Pritchard & Smith 1988, Lyne, Pritchard & Smith 1993); for PSR B 1509-58, n= 2.837 ± 0.001 (Kaspi et al. 1994); for PSR B 0540-69, n= 2.04 ± 0.02 (Manchester & Peterson 1989, Nagase et al. 1990, Gouiffes, Finley & Ögelman 1992); for pulsar J 1119-6127, n= 2.91 ± 0.05 (Camilo et al. 2000); for pulsar J 1846-0258, n=2.65 ± 0.01 (Livingstone et al. 2006). For the Vela pulsar a long term (secular) braking index of 1.4 ± 0.2 was reported (Lyne et al. 1996). This value was extracted with certain assumptions for connecting fiducial epochs across a timing history dominated by glitches and interglitch response.

For pulsars with moderate ages, $\sim 10^5$ yr, anomalous braking indices have values of order $\pm10^2$. These are not noise artefacts. Rather, such braking indices can be understood as part of the neutron star's secular dynamics. The interglitch recovery of pulsars extending through observation time spans may yield positive anomalous braking indices, while negative anomalous braking indices can be explained by the occurrence of an unobserved glitch causing a negative step $\Delta\dot{\Omega}$ in the spindown rate (as typically observed with resolved glitches), between the different measurements of $\dot{\Omega}$ (Johnston & Galloway 1999). In this work we show that all pulsars with anomalous $\ddot{\Omega}$ measurements, including all the pulsars with observed glitches as well as post glitch or interglitch $\ddot{\Omega}$ values (Shemar & Lyne 1996, Wang et al. 1999) obey the same scaling between $\ddot{\Omega}$ and glitch parameters (Alpar 1998) as in the models developed for the Vela pulsar glitches (Alpar et al. 1993).

2 The Model for Glitches and Interglitch Dynamics

In the absence of evidence that the pulsar electromagnetic torque changes at a glitch, and with the established impossibility of explaining the large ($\Delta\Omega/\Omega > 10^{-7}$) and frequent (intervals ~ 2 yrs) Vela pulsar glitches with starquakes, the glitch is modelled as a sudden angular momentum exchange between the neutron star crust and an interior component,

$$I_c\Delta\Omega_c = I_s\delta\Omega = (I_A/2 + I_B)\delta\Omega. \tag{3}$$

Here $\Delta\Omega_c$ is the observed increase of the crust's rotation rate at the glitch. I_c is the effective moment of inertia of the crust, including all components of the star dynamically coupled to the crust on timescales shorter than the resolution of the glitch event. The observations imply that I_c includes practically the entire moment of inertia of the star, and the theory of the dynamical coupling mechanisms of the neutron star core (Alpar, Langer & Sauls 1984) provides an understanding of this by furnishing crust-core coupling times shorter than the resolution of glitch observations.

$\delta\Omega$ describes the decrease in the rotation rate of the pinned superfluid at the glitch. I_A and I_B are parts of the superfluid's effective moment of inertia I_s associated with different dynamical behaviour. The vortex lines are the discrete carriers of the superfluid's angular momentum. Vortex lines under pinning forces respond to the driving external pulsar torque, as this torque makes the normal crust lattice spin down.

At finite temperature, the motion of these vortices against the pinning energy barriers is made possible by thermal activation. A different possibility, operating even at T = 0, is quantum tunneling. It can be shown easily that if vortices unpinned in a glitch are unpinned at a uniform density throughout the creep regions of moment of inertia I_A, then the angular momentum transfer from these regions to the normal crust is $I_A\delta\Omega/2$, as in the right hand side of Eq. (3) (Alpar et al. 1984a, 1993).

The continuous spindown between glitches is governed by:

$$I_c\dot{\Omega}_c = N_{ext} + N_{int} = N_{ext} - I_A\dot{\Omega}_s, \tag{4}$$

where N_{ext} is the external torque on the neutron star, and N_{int} is the internal torque coupling the superfluid to the "effective crust" with moment of inertia $I_c \cong I$.

In a cylindrically symmetric situation the spindown rate of the superfluid is proportional to the mean vortex velocity in the radial direction, which in turn is determined by the lag $\omega = \Omega - \Omega_c$ between the superfluid and crust rotation rates:

$$\dot{\Omega}_s = -\frac{2\Omega_o}{r}V_r(\omega). \tag{5}$$

As the glitch imposes a sudden change in ω, it will offset the superfluid spindown and therefore the observed spindown rate of the crust, according to Eq. (4). The glitch is followed by transient relaxation processes in which the crust rotation frequency and spindown rate relax promptly as an exponential function of time (Alpar et al. 1984a,b). It is the long term interglitch relaxation of the spindown rate, after the transients are over, that determines the interglitch behavior of the observed crust spindown rate.

Labelling the moment of inertia associated with long term offset in spindown rate with I_A, from Eq.(4) we have

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} = \frac{I_A}{I}. \tag{6}$$

contribution of the regions I_A to the glitch in the rotation frequency is $I_A\delta\Omega/(2I)$. Together the contributions of the 'resistive' (continuous vortex current) regions A and the 'capacitive' vortex trap (accumulation) regions B give Eq. (3).

4 Anomalous Braking Indices

Braking indices were measured, at various degrees of accuracy as the data permitted, from 8 (excluding the Crab and Vela pulsars) out of 18 glitching pulsars studied by Lyne, Shemar & Graham-Smith (2000), and from 9 (excluding the Vela pulsar) out of 11 glitching southern pulsars studied by Wang et al. (2000). Some of these pulsars are common to both surveys. We exclude the Crab and Vela pulsars in the present work because detailed postglitch and interglitch data and fits exist for these pulsars; indeed the long term interglitch behaviour of the Vela pulsar provides the prototype dynamical behaviour that we are searching for in pulsars older than the Vela pulsar. For three pulsars common to both surveys, PSR J 1341-6220, PSR J 1709-4428 and PSR J 1801-2304, Wang et al. (2000) quote $\ddot{\Omega}$ measurements, while Lyne, Shemar & Graham-Smith (2000) quote upper limits to $\ddot{\Omega}$ for two of these pulsars. Thus there are now published $\ddot{\Omega}$ measurements for 14 out of 23 glitching pulsars excluding the Crab and Vela pulsars. We have tabulated 10 of these according to the significance of error bars.

In addition, Johnston & Galloway (1999) have obtained braking indices for 20 pulsars to demonstrate the method they proposed, applying Eq. (2) to rotation frequency and spindown rate measurements at two different epochs. These pulsars were not known glitching pulsars, and they were not observed to glitch during these observations. Anomalous braking indices were found for all 20 pulsars, with negative values in 6 pulsars and positive values in the rest. Of the data in the Johnston and Galloway sample, we shall take into consideration those data sets for which the quoted errors in the braking index are less than the quoted value, so that there is no ambiguity in the sign of the braking index. With this criteria, we study 18 pulsars, 5 with negative and 13 with positive braking indices. From two of these pulsars Johnston and Galloway reported two distinct data sets. Thus our sample contains 20 determinations of the braking index from 18 pulsars. Johnston & Galloway (1999) have interpreted the positive anomalous braking indices as due to interglitch recovery, without evoking a specific model. They interpreted the negative braking indices as reflecting an unresolved glitch during their observation time spans. All glitches result in long term decrease of the spindown rate, i.e. a negative step, an increase in the absolute value, of the rate of spindown. Since the pulsars were not monitored continuously, a glitch occurring between two timing observations would lead to a negative $\ddot{\Omega}$ inference, equivalent to a negative braking index.

4 Braking Indices of Pulsars Not Observed to Glitch

All glitches bring about a sudden negative change $\Delta\dot{\Omega}$ in $\dot{\Omega}$, that is, a fractional increase $\Delta\dot{\Omega}/\dot{\Omega}$ by 10^{-3}-10^{-2} in the spindown rate. If the unresolved glitch happens in a timespan of length t_i, the offset $\Delta\dot{\Omega}$ in the spindown rate will mimic a negative second derivative of the rotation rate, $\ddot{\Omega} = \Delta\dot{\Omega}/t_i$.

Let us first elaborate on the statistical analysis of the negative braking index pulsars as those suffering an unobserved glitch during a gap within the timespan of the observations, following the analysis of Johnston & Galloway (1999) and using, as these authors did, the statistical glitch parameters of Alpar & Baykal (1994). The probability that pulsar i has one glitch during the timespan t_i of the observations is given by the Poisson distribution $P(1;\lambda_i) = \lambda_i\, exp(-\lambda_i)$ where the parameter λ_i is given by $\lambda_i = \frac{t_i}{t_{g,i}}$ and $t_{g,i}$ is the time between glitches for pulsar i.

To derive $t_{g,i}$ with Eq.(8), one needs to know the decrease $\delta\Omega_i$ in superfluid rotation rate at the previous glitch. In this sample of pulsars from which glitches have not been observed we estimate the value of $\delta\Omega_i$ by making two alternative hypotheses about the constancy of average glitch parameters among pulsars older than the Vela pulsar and equating the parameters to their average values for the Vela pulsar glitches. Under the first hypothesis $\delta\Omega$ is assumed to be constant for all pulsar glitches, and is set equal to $<\delta\Omega>_{Vela}$, the average value inferred for the Vela pulsar glitches:

$$\delta\Omega_i{}^{(1)} = <\delta\Omega>_{Vela} \tag{7}$$

$$\lambda_i{}^{(1)} = \frac{t_i|\ddot{\Omega}|_i}{<\delta\Omega>_{Vela}}. \tag{8}$$

Under the second hypothesis, $\delta\Omega/\Omega$ is assumed to be constant for all glitches of pulsars older than the Vela pulsar. Johnston & Galloway (1999) adopted this hypothesis, taking the value estimated by Alpar & Baykal (1994) from glitch statistics, which agrees with the range of values of $\delta\Omega/\Omega$ inferred for the Vela pulsar glitches,

$$<\delta\Omega/\Omega>_i^{(2)} = 1.74 \times 10^{-4} \tag{9}$$

$$\lambda_i^{(2)} = 5.75 \times 10^3\frac{t_i|\dot{\Omega}|_i}{\Omega_i} = 2.87 \times 10^{-3}\frac{t_i}{\tau_{i,6}}. \tag{10}$$

Here t_i is in years and $\tau_{i,6}$ is the dipole spindown age of pulsar i in units of 10^6 years. Table 1 gives the values of $\lambda_i^{(1)}$ and $\lambda_i^{(2)}$. The corresponding probabilities P (1 ; λ_i) for an (unobserved) glitch to fall within the observation timespan devoted to pulsar i, or, equivalently, pulsar i mimicking a negative second derivative, are quite low for either hypothesis, while the probabilities P (0 ; λ_i) $\cong$ 1 for no glitch occurring within the observation timespan of pulsar i, or, equivalently, a positive anomalous braking index being measured for pulsar i. The probability that 5 out of the 18 pulsars' 20 data sets sampled have had unresolved glitches within the observation timespans, so that they have negative anomalous second derivatives, is given by

$$P(5;\lambda^{(j)}) = (\lambda^{(j)})^5\, exp(-\lambda^{(j)})/5! \tag{11}$$

where

$$\lambda^{(j)} = \sum_{i=1}^{20}\lambda_i{}^{(j)} \tag{12}$$

for the hypotheses j = 1, 2. The index in this runs over all data sets, since 2 of the 8 pulsars have two independent data sets each in the sample of Johnston & Galloway (1999). We find that

$$\lambda^{(1)} = 1.33 \tag{13}$$

$$P(5;\lambda^{(1)}) = 0.0092 \tag{14}$$

$$\lambda^{(2)} = 3.11 \tag{15}$$

$$P(5;\lambda^{(2)}) = 0.11 \quad . \tag{16}$$

6 Conclusions

This means that hypothesis (2) is likely to be true, since it gives a total expected number of glitches falling within observation timespans to be 3.11 against the number 5 implied by this interpretation of negative braking indices, as Johnston & Galloway (1999) noted. With hypothesis (1) the expected number of glitches is $\lambda^{(1)} = 1.33$ and 5 glitches within observation timespans has a lower $P(5;\lambda^{(1)}) = 0.0092$ probability so this hypothesis is not favored. The same conclusion was reached by Alpar & Baykal (1994) on the basis of statistics of large pulsar glitches: with the hypothesis (1), that $\delta\Omega$ is roughly constant in all pulsars older than Vela, the statistics implied $<\delta\Omega>= 0.0188$, which does not agree with $<\delta\Omega>_{Vela}= 0.0094$

References

Alpar M.A., Anderson P.W., Pines D., Shaham J., 1984a, ApJ., 276, 325

Alpar M.A., Anderson P.W., Pines D., Shaham J., 1984b, ApJ., 278, 791

Alpar, M.A., Baykal, A., 1994, MNRAS, 269, 849

Johnston S., Galloway D., 1999, MNRAS, 306, L50