FORCE-FREE MAGNETOSPHERE OF AN ALIGNED ROTATOR

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Abstract We explore properties of stationary force-free magnetosphere of an aligned rotator in the most general case, taking into account differential rotation of the open magnetic field lines. We conclude, that most probably the electromagnetic cascades in the polar cap of pulsar are non-stationary and the size of the corotating region is less that the radius of the Light Cylinder.

Pulsar Equation

Structure of the force-free magnetosphere of an aligned rotator can be described by a set of solutions of the so-called pulsar equation, which in cylindrical coordinates (x, ϕ, z) has the form

$$(\beta^2 x^2 - 1)(\partial_{xx}\psi + \partial_{zz}\psi) + \frac{\beta^2 x^2 + 1}{x}\partial_x\psi - S\frac{dS}{d\psi} + x^2\beta\frac{d\beta}{d\psi}(\nabla\psi)^2 = 0\,. \quad (1)$$

ψ and S are normalized magnetic flux and poloidal current correspondingly. Magnetic field is expressed through these functions as

$$\mathbf{B} = \frac{\mu}{R_{\mathrm{LC}}^3}\left(\frac{\nabla\psi \times \mathbf{e}_\phi}{x} + \frac{4\pi}{c}\frac{S}{x}\,\mathbf{e}_\phi\right)\,. \quad (2)$$

where μ is the magnetic moment of the neutron star (NS). All coordinates are normalized to $R_{\mathrm{LC}}^{\mathrm{cor}} \equiv c/\Omega$, the radius of the Light Cylinder for corotating plasma. $\beta \equiv \Omega_{\mathrm{F}}/\Omega$ - is the ratio of the angular velocity of magnetic field lines Ω_{F} normalized to the angular velocity of the neutron star Ω. Angular velocity of rotation of magnetic field lines is given by

$$\Omega_{\mathrm{F}} = \Omega(1 + \frac{\partial V}{\partial\psi})\,. \quad (3)$$

where V is the non-corotational electric potential. Angular velocity of open magnetic field lines differs from the the angular velocity of the NS due to existence of an accelerating electric field in the polar cap of pulsar.

At the Light Cylinder (LC), i.e. in points with coordinates (x_{LC}, z) such that

$$x_{\mathrm{LC}}(z)\cdot\beta\,[\psi(x_{\mathrm{LC}}, z)] = 1 \quad (4)$$

the pulsar equation has singularity and it reduces to

$$2\beta\,\partial_x\psi + \frac{1}{\beta}\frac{d\beta}{d\psi}(\nabla\psi)^2 = S\frac{dS}{d\psi}\,. \quad (5)$$

Each *smooth* solution of eq. (1) must satisfy eq. (5) at LC. This is possible only if function S satisfies eq. (5).

Force-free-magnetosphere of an aligned rotator has two physical degrees of freedom: i) the size of the closed field line zone, and ii) the distribution of angular velocity of open magnetic field lines $\beta(\psi)$.

$\beta(\psi)$ is determined by the local electrodynamics of the polar cap.

The current density distribution in the pulsar magnetosphere for a given size of the closed field line zone x_0 and for a given angular velocity distribution $\beta(\psi)$ is a **fixed function**. It is obtained in course of solving of the pulsar equation, from eq. (5).

Solution method

We assume Y-configuration of the magnetosphere, i.e. the existence of an equatorial current sheet with the return current. Equation (1) is solved numerically in a domain $x_{\mathrm{NS}} \le x \le x_{\max}$, $z_{\mathrm{NS}} \le z \le z_{\max}$

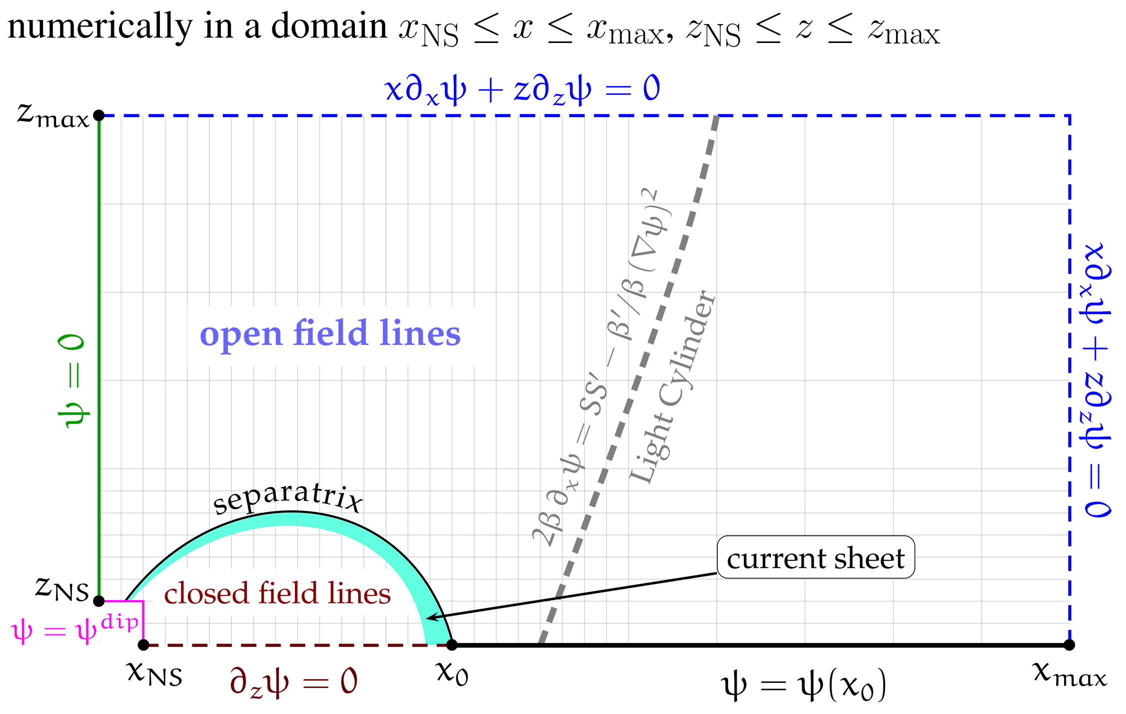

FIGURE 1: *Calculation domain and boundary conditions*

Boundary conditions are shown in Fig. 1. Equation (1) is solved using multigrid FAS scheme. As a smoother we used Gauss-Seidel scheme.

At each iteration step we find position of the LC by solving numerically the equation (4) by the Newton method for each z-axis grid point z_j, and find poloidal current function $SS' \equiv S\,(dS/d\psi)$ from the equation (5). Then we use piece-polynomial interpolation for SS' and calculate $SS'(x,z) = SS'\,[\psi(x,z)]$ in each domain point.

The return current flowing along the last closed magnetic field line is smeared over the region $[\psi_0 - d\psi, \psi_0]$. β in the current sheet smoothly changes to the value $\beta = 1$ in the closed field line zone.

Main Results

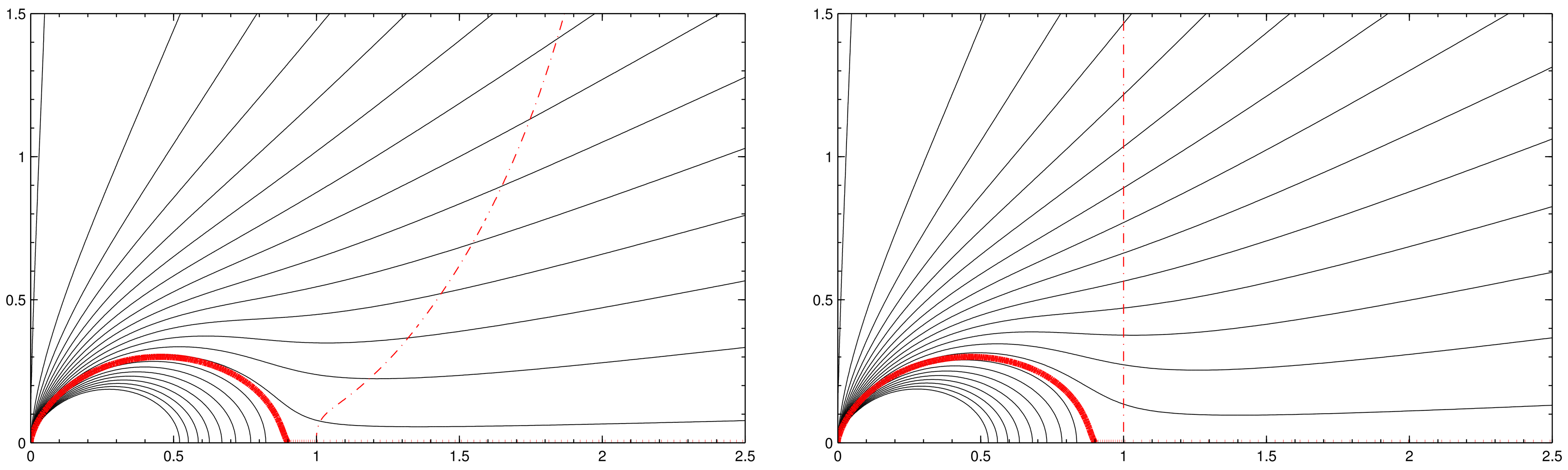

FIGURE 2: *Structure of the pulsar magnetosphere:* ***left*** *– for $x_0 = 0.9$ and such distribution of angular velocity of open field lines $\beta(\psi) = \bar\beta(\psi)$, that the current density in the polar cap of pulsar is nearly constant.* ***right*** *– for $x_0 = .9$ and $\beta \equiv 1$. The magnetic field lines are shown by thin black lines for the same values of ψ in both figures. The last open magnetic field line is shown by the thick red line. The Light Cylinder is shown by the dot-dashed line.*

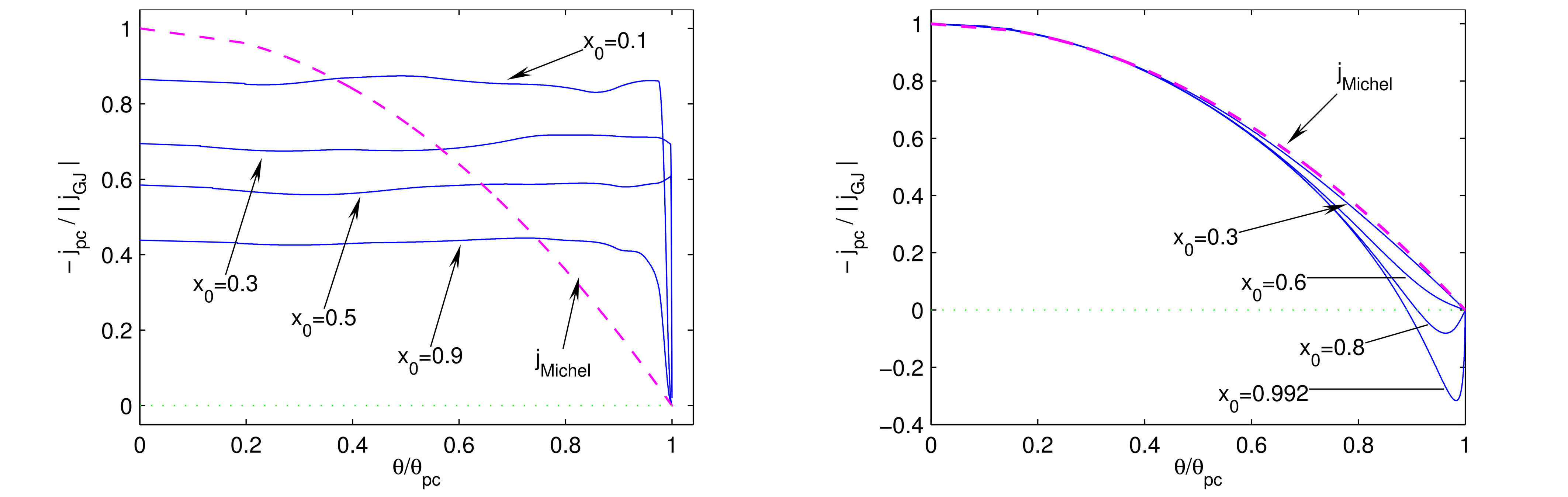

FIGURE 3: *Poloidal current density in the polar cap of pulsar normalized to the Goldreich-Julian current density for different values of x_0:* ***left*** *– for variable $\beta(\psi) = \bar\beta(\psi)$ producing nearly constant current density, $j_{\mathrm{pc}} \simeq \bar j$.* ***right*** *– for $\beta \equiv 1$. Michel current density is shown by the dashed line.*

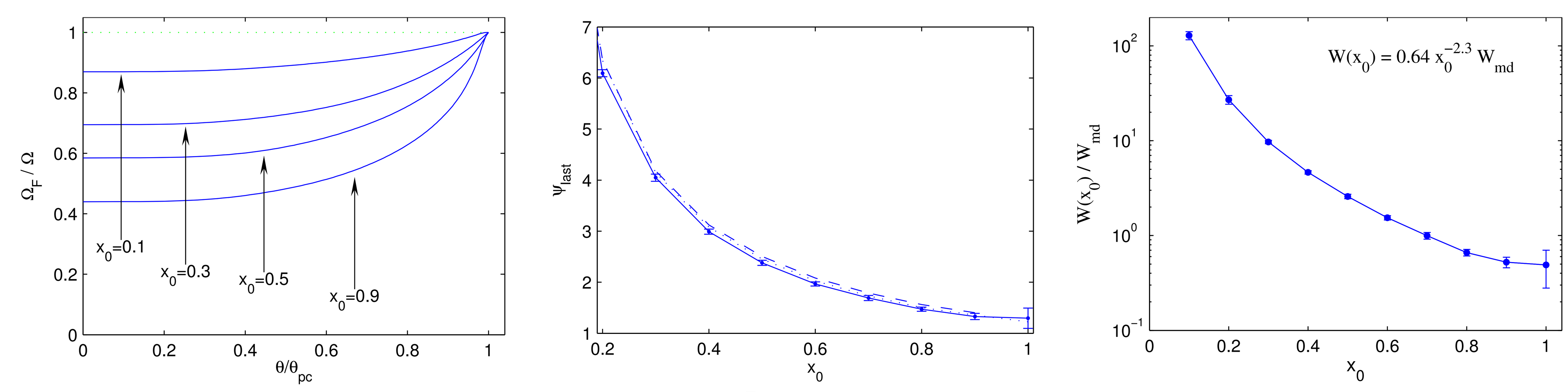

FIGURE 4: *From left to right:* ***(1)*** *– angular velocity of rotation $\bar\beta$ of the open field lines for different x_0 as a function of the colatitude in the polar cap of pulsar. $\bar\beta(\psi)$ shown here correspond to the current densities shown in Fig. 3 (left);* ***(2)*** *– the value of normalized magnetic flux function corresponding to the last open magnetic field line as function of x_0: solid line – for $\beta(\psi) = \bar\beta(\psi)$ shown in panel (1), dashed line – for $\beta(\psi) \equiv 1$, dotted line – for $\beta \equiv \min(\bar\beta(\psi))$;* ***(3)*** *– energy losses of pulsar with almost constant current density in the polar cap, normalized to the magnetodipolar energy losses as a function of x_0;*

- For $\beta(\psi) \equiv 1$ in configurations with $x_0 > 0.6$ there is a *volume* return current flowing along the open magnetic field lines, which makes however only a small part of the whole return current. The current density (in units of j_{GJ}) close to the polar cap boundary increases with increasing of x_0. The current density does not exceed the corresponding Michel current density.
- Current density can be made nearly constant over the polar cap of pulsar ($j_{\mathrm{pc}} \simeq \bar j \equiv \mathrm{const}$) by adjusting angular velocity of open field lines ($\beta(\psi) \equiv \bar\beta(\psi)$), Fig. 3 (left).
- $\bar j$ increases with increasing of x_0. However, $\bar j < j_{\mathrm{GJ}}$ for any x_0, see Fig. 3 (left).
- Deviation of $\bar\beta(\psi)$ from 1 decreases with decreasing of x_0, i.e. in configurations with larger x_0 the corresponding potential drop in the polar cap must be larger, Fig. 4 [1]
- Configurations with constant and variable β's ave similar even if the corresponding current density distributions are significantly different, see Fig. 2.
- Values of ψ corresponding to the last open magnetic field line for configurations with $\beta \equiv 1$ and for configurations with $\beta(\psi) \equiv \bar\beta(\psi)$ ave very close to each other for any x_0, Fig. 4 [2].
- Energy losses of the pulsar increase with decreasing of x_0, Fig. 4 [3].
- Each solution has been checked for applicability of the force-free condition $E < B$. In none of them this condition is violated.
- The total energy of the electromagnetic field in the magnetosphere $\Xi = \int_{Vol}(B^2 + E^2)/(8\pi)\,dV$ decreases with increasing of x_0.

Conclusions

- Electromagnetic cascades in the polar cap of pulsar should be non-stationary.
- The size of the closed field line zone should be less than the Light Cylinder radius, $x_0 < 1$
- The magnetosphere of pulsar should evolve with time, i.e the relative size (in R_{LC}) of the closed field line zone should change. This will result in pulsar breaking index different from 3.
- The configuration of the magnetosphere is mostly determined by the size of the corotating region x_0.

References: [1] A. N. Timokhin, 2006, MNRAS, Vol. **368**, p. 1055; [2] A. N. Timokhin, 2006, to appear in ApSS, astro-ph/0607165; [3] A. N. Timokhin, 2006, in preparation