Poster Presentation

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External Electromagnetic Fields of Slowly Rotating Relativistic Magnetized NUT Stars

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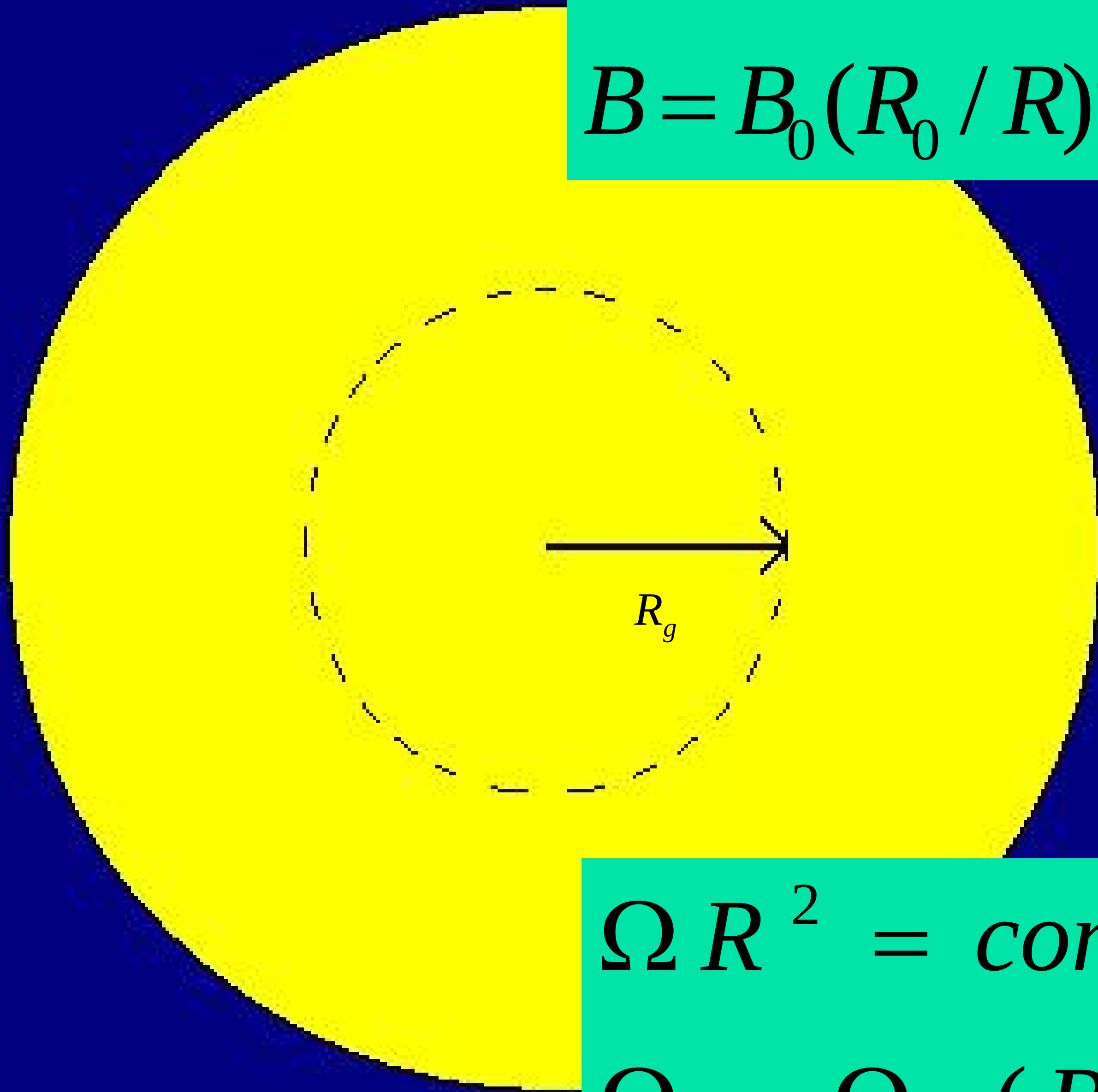
Plan of talk

- Basic assumptions and approach
- Electromagnetic Fields of Slowly Rotating Axial Symmetric Magnetized NUT Star
- Conclusion

$$BR^2 = const \Rightarrow$$

$$B = B_0 (R_0 / R)^2 \propto 10^{12} G$$



$$\Omega R^2 = const \Rightarrow$$

$$\Omega = \Omega_0 (R_0 / R)^2$$

$$R \sim 10km, R_g = \frac{2GM}{c^2 r} \sim 5km, (1 - \frac{2GM}{c^2 r}) \sim 0.5$$

- GR effects are important for compact objects

- Immense difficulty of simultaneously solving the Maxwell eqs and the highly nonlinear Einstein eqs

$$e^{\alpha\beta\mu\nu} F_{\beta\mu,\nu} = 0,$$

$$F^{\alpha\beta}{}_{;\beta} = 4\pi J^{\alpha}$$

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \kappa T_{\alpha\beta},$$

$$T_{\alpha\beta} = T_{(em)\alpha\beta} + T_{(G)\alpha\beta}$$

APPROACH

EMF are considered in a given background geometry

$$R_{\alpha\beta} - g_{\alpha\beta} R/2 = \kappa T_{\alpha\beta}$$

$$T^{\alpha\beta} = T^{\alpha\beta}_{(matter)} + T^{\alpha\beta}_{(em)} \text{ ¼ } T^{\alpha\beta}_{(matter)}$$

$\rho_{(em)} c^2$ ¼ $B^2/8\pi$) $\rho_{(em)}$ ¼ 10^3 g/cm^3 if B s 10^{12} G

$\rho_{(em)}$ ¿ $\rho_{(matter)}$ ¼ 10^{14} g/cm^3 consequently $T^{\alpha\beta}_{(em)}$ ¿ $T^{\alpha\beta}_{(matter)}$

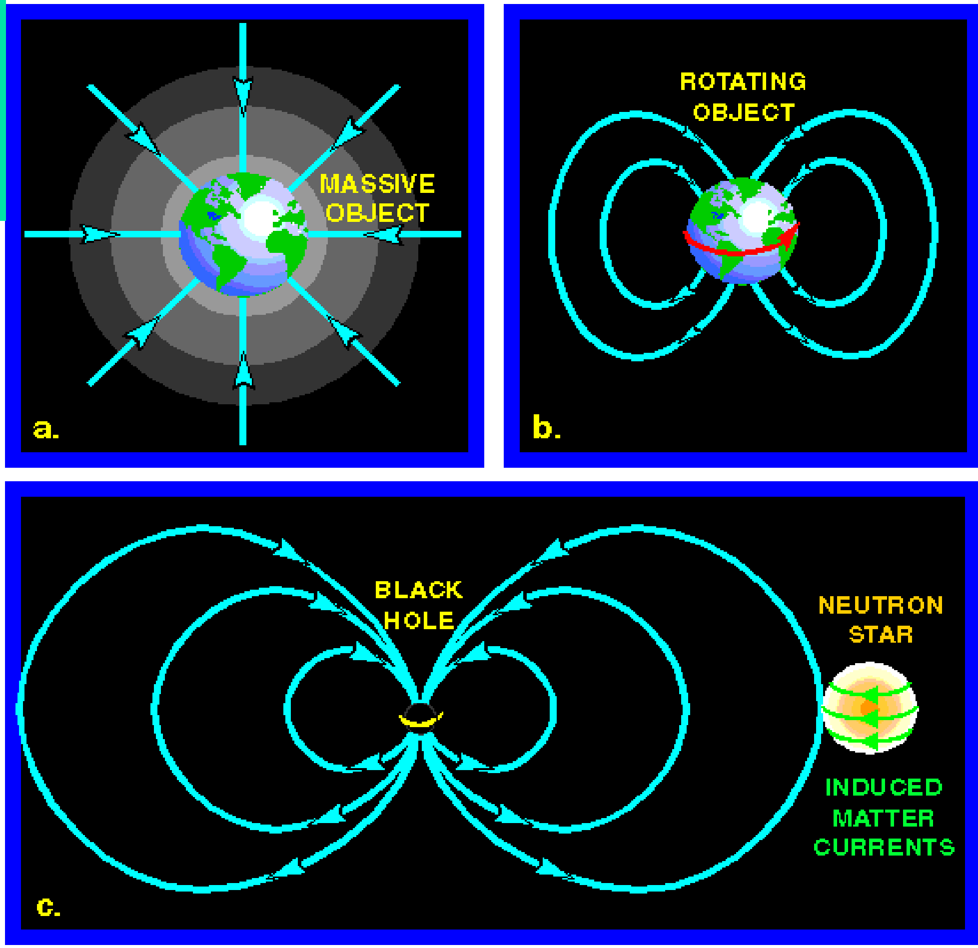
Slow rotation approximation: comparing the neglected terms with the lower order one gives ratios

$$R^3\Omega^2/GM < 10\%$$

even for fastest rotating stars (PSR J1748-2446ad & PSR 1937+214)

$$m\frac{d\vec{v}}{dt} = m(\vec{E}_g + \frac{1}{c}\vec{v}\times\vec{B}_g),\ \vec{E}_g = -\frac{GM}{r^2}\vec{r}$$

$$\vec{B}_g = \frac{2G}{c}[\frac{\vec{J} - 3(\vec{J}\cdot\vec{r})\vec{r}}{r^3}]$$



METRIC
OF SLOWLY ROTATING NUT STAR

$$ds^2 = -N^2 dt^2 + N^{-2} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 - 2\left[\omega(r) r^2 \sin^2\theta + 2lN^2 \cos\theta\right] dt d\phi$$

$$\frac{B^2}{8\pi\mu c^2} \simeq 1.6 \times 10^{-6} \left(\frac{B}{10^{15}\ \mathrm{G}}\right)^2 \left(\frac{1.4 M_\odot}{M}\right) \left(\frac{R}{15\ \mathrm{km}}\right)^3$$

$$\frac{E^2}{8\pi\mu c^2} \simeq 0.2 \times 10^{-20} \left(\frac{E}{3 \times 10^{10}\ \mathrm{V/cm}}\right)^2 \left(\frac{1.4 M_\odot}{M}\right) \left(\frac{R}{15\ \mathrm{km}}\right)^3$$

ω(r)´ 2aM/r³

ZAMO OBSERVER:

$$(u^{\alpha})_{\rm ZAMO} \equiv N^{-1}\left(1,0,0,\omega + \frac{2l\cos\theta}{r^2\sin^2\theta}N^2\right); \qquad (u_{\alpha})_{\rm ZAMO} \equiv N\left(-1,0,0,0\right).$$

$$N \equiv \left(1 - \frac{2M}{r}\right)^{1/2}$$

Monopolar configuration for MF

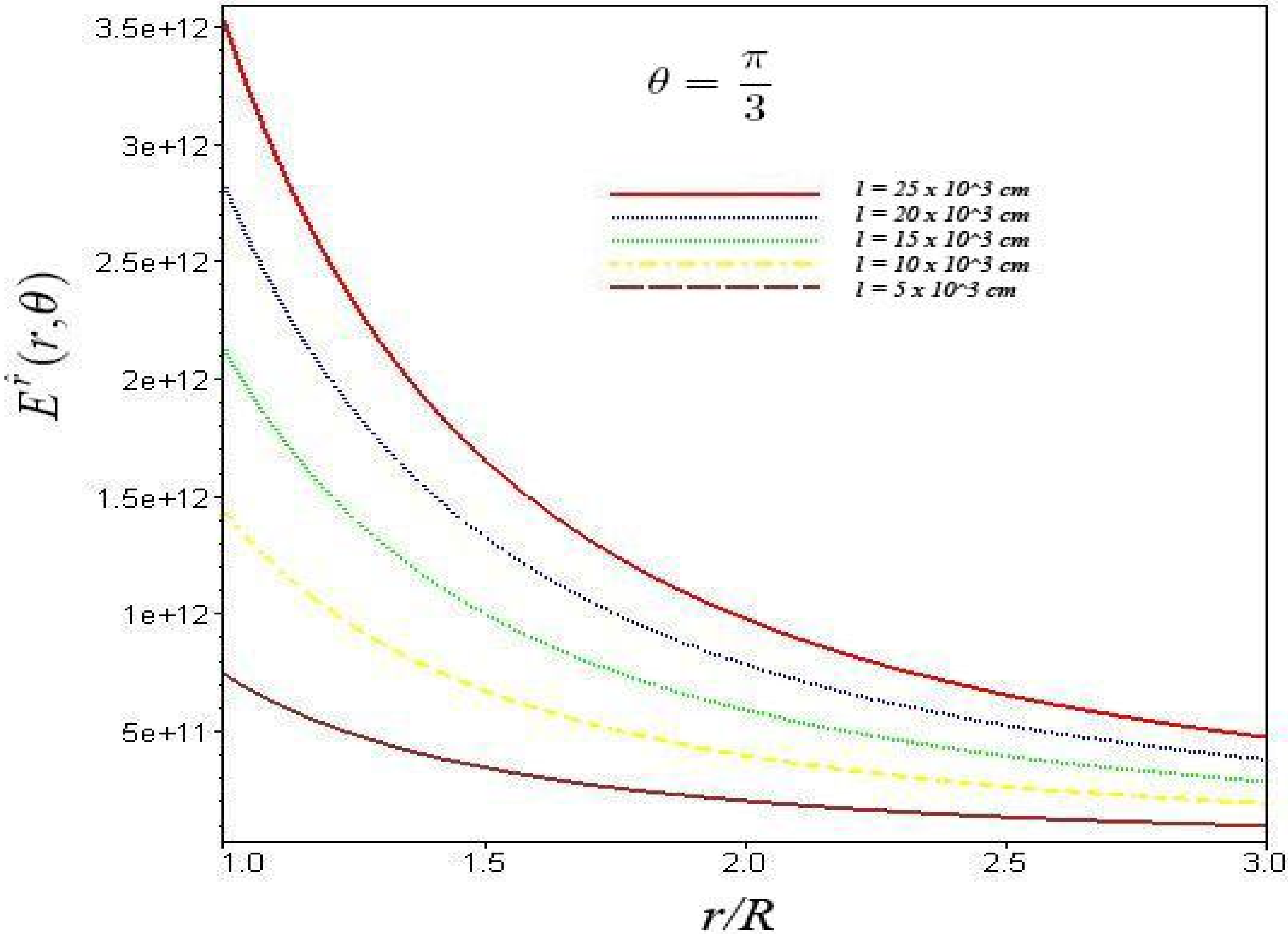
$$B^r = \frac{\mu_0}{r^2}$$

$$E^{\hat{r}}_{,\theta} - (rNE^{\hat{\theta}})_{,r} - \frac{3\omega\mu_0}{r}\sin\theta - \frac{4\mu_0 l}{r^3} ctg\theta = 0$$

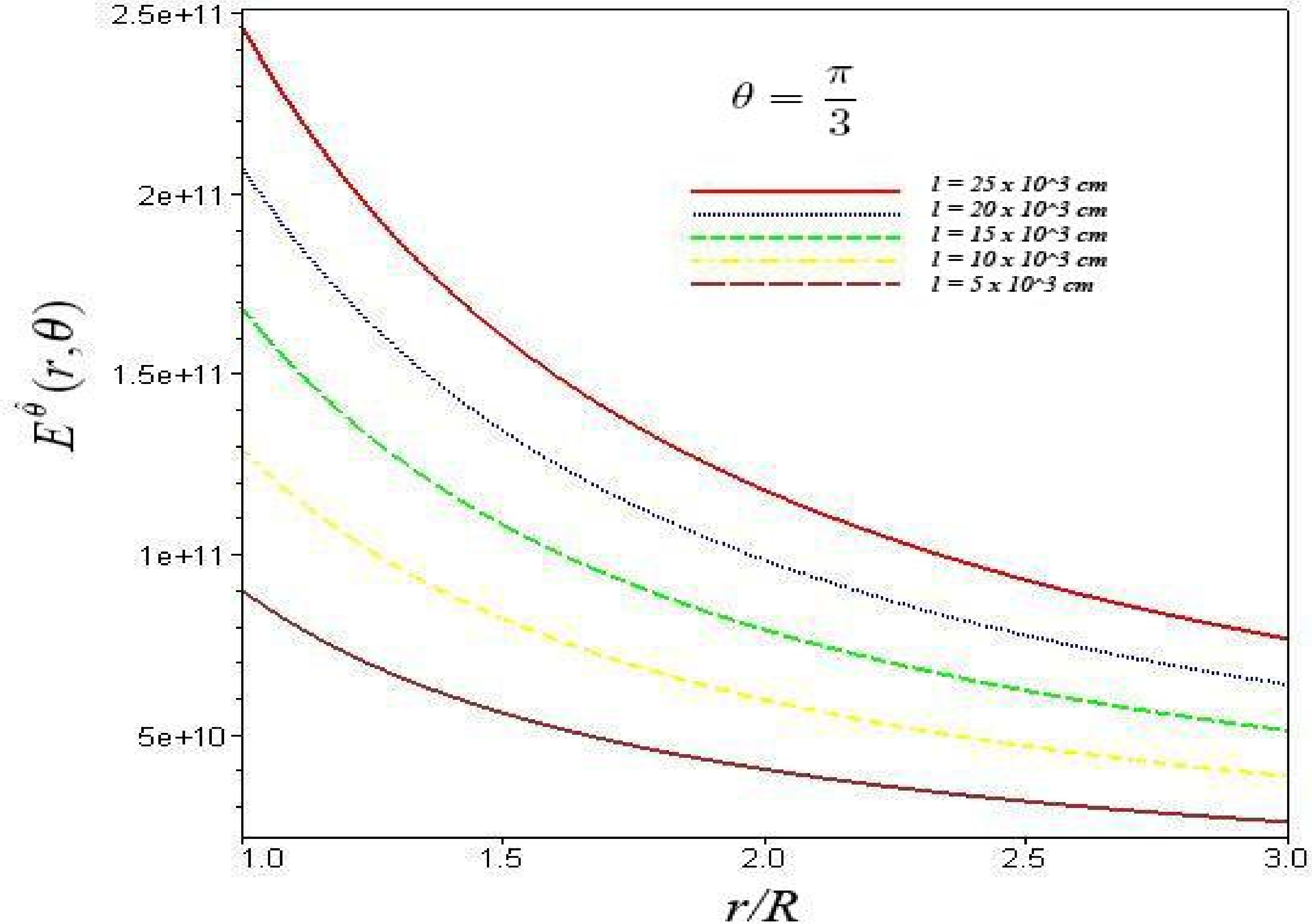
$$\sin\theta\,(r^2 E^{\hat{r}})_{,r} + \frac{r}{N}(\sin\theta E^{\hat{\theta}})_{,\theta} = 0$$

$$E^{\hat{r}}(r,\theta)=\frac{\Omega R\mu_0}{r^2}-\left(\frac{\omega\mu_0}{M}\right)\cos\theta-\left(\frac{\mu_0 l}{2M^2}\right)\frac{1}{r}\left[1+\frac{2M}{r}\ln\frac{r}{2M}\right]$$

$$E^{\hat{\theta}}(r,\theta)=-N\frac{\mu_0}{2M}\left\{\omega\sin\theta+\frac{l}{2Mr}\left(1+\frac{2M}{r}\right)ctg\theta\right\}$$



$\theta = \frac{\pi}{3}$
l = 25 x 10^3 cm
l = 20 x 10^3 cm
l = 15 x 10^3 cm
l = 10 x 10^3 cm
l = 5 x 10^3 cm
3.5e+12
3e+12
2.5e+12
2e+12
1.5e+12
1e+12
5e+11
$E^{\hat{r}}(r,\theta)$
1.0
1.5
2.0
2.5
3.0
r/R



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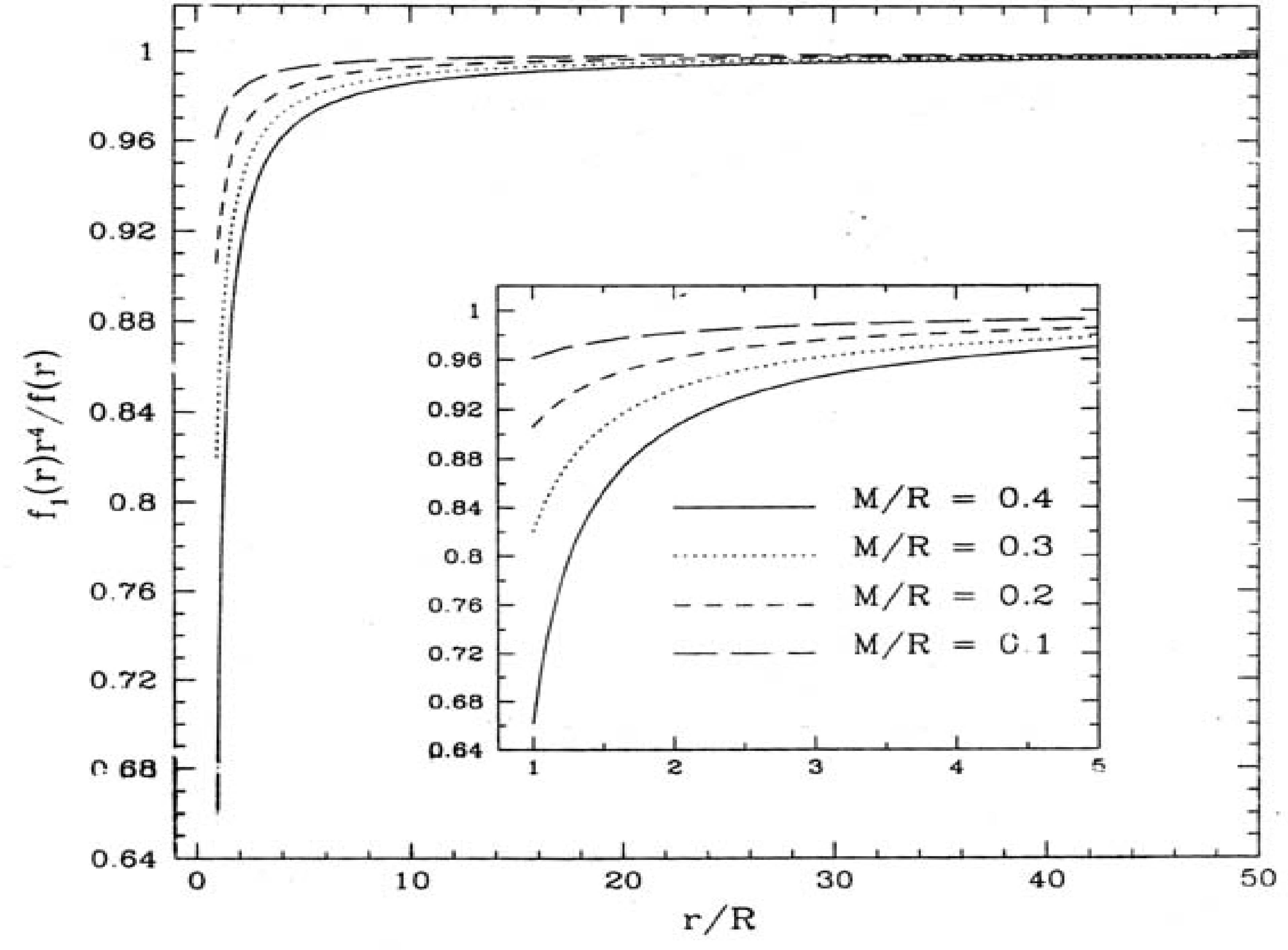
$E^{\hat{\theta}}(r,\theta)$
r/R
2.5e+11
2e+11
1.5e+11
1e+11
5e+10
1.0
1.5
2.0
2.5
3.0

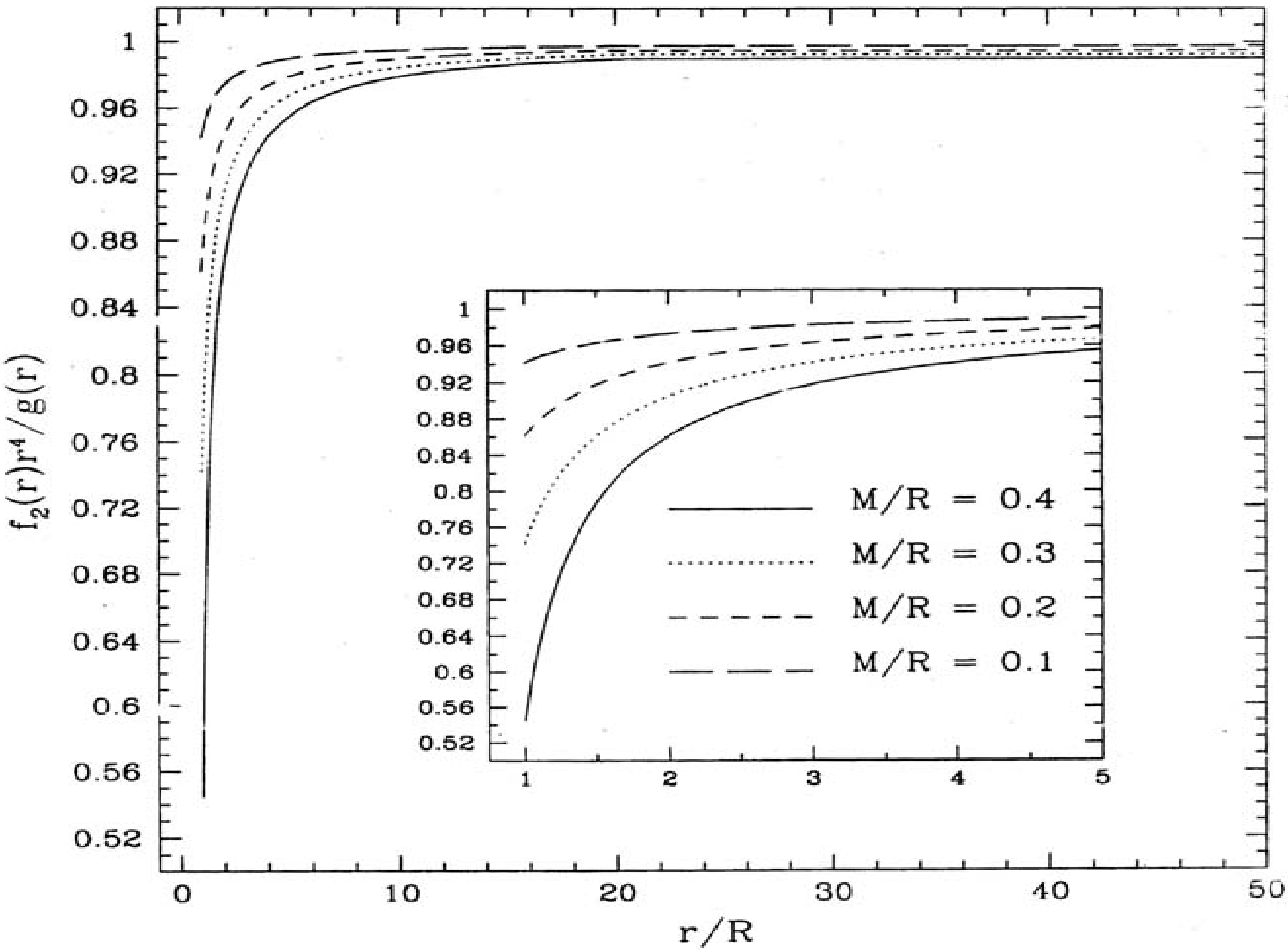
DIPOLAR MAGNETIC FIELD

$$B^r = -\frac{3\mu}{4M^3}[\ln N^2 + \frac{2M}{r}(1+\frac{M}{r})]\cos\theta,$$

$$B^\theta = \frac{3\mu N}{4M^2 r}[\frac{r}{M}\ln N^2 + \frac{1}{N^2} + 1]\sin\theta.$$



$f_1(r)r^4/f(r)$
r/R
M/R = 0.4
M/R = 0.3
M/R = 0.2
M/R = 0.1



1
0.96
0.92
0.88
0.84
0.8
0.76
0.72
0.68
0.64
0.6
0.56
0.52
$f_2(r)r^4/g(r)$
0
10
20
30
40
50
r/R
M/R = 0.4
M/R = 0.3
M/R = 0.2
M/R = 0.1
1
2
3
4
5

$$E^{\hat{r}} = \left\{ \frac{15\omega r^3}{16M^5c} \left\{ C_3 \left[\left(3 - \frac{2r}{M}\right) \ln N^2 + \frac{2M^2}{3r^2} + \frac{2M}{r} - 4 \right] + \frac{2M^2}{5r^2} \ln N^2 + \frac{4M^3}{5r^3} \right\} \right.$$

$$\left. + \frac{\Omega}{6cR^2} C_1 C_2 \left[\left(3 - \frac{2r}{M}\right) \ln N^2 + \frac{2M^2}{3r^2} + \frac{2M}{r} - 4 \right] \right\} \left(3\cos^2\theta - 1\right)\mu - \frac{2\mu l}{3Mr^3} \cos\theta \ ,$$

$$E^{\hat{\theta}} = -\left\{ \frac{45\omega r^3}{16M^5c} N \left\{ C_3 \left[\left(1 - \frac{r}{M}\right) \ln N^2 - 2 - \frac{2M^2}{3r^2N^2} \right] + \frac{4M^4}{15r^4N^2} \right\} \right.$$

$$\left. + \frac{\Omega}{2cR^2} C_1 C_2 N \left[\left(1 - \frac{r}{M}\right) \ln N^2 - 2 - \frac{2M^2}{3r^2N^2} \right] \right\} \left[2\sin\theta\cos\theta\right]\mu - \frac{\mu l}{3Mr^3} \sin\theta \ .$$

CONCLUSION

- E/M Fields of Rotating Magnetized NUT stars in General Relativity:

GR Effects Resulting from Dragging of Inertial Frames $\omega = 2aM / r^3$

Monopolar Part GM/c^2R

and NUT parameter l

are Very Important for EF

For MF only mass of star plays a role