# Pulse Shape Modeling for accreting msec Pulsars

Denis Leahy, U. Calgary

Collaborators:

Sharon Morsink, Coire Cadeau, U. Alberta

# Outline

- Motivation: accreting ms pulsars
- Properties: rapid rotation, general relativity
- Methods: exact and approximate calculations
- Some results: exact light curves; approximate light curve fits to observations

# Motivation: accreting ms pulsars

Rotating neutron star with surface hot spot yields pulsations

Aim: to model the pulsations to obtain N* parameters

Circular antipodal spots in Schwarzschild: Pechenick, Ftaclas & Cohen, 1983, ApJ, 274, 876

Main application has been to slowly rotating X-ray pulsars (Leahy 2004 ApJ 613, 517; Kraus et al 2003 ApJ 590, 424 and references therein)

Application to ms pulsars recent (Miller & Lamb 1998 ApJ 499 L37, Poutanen & Gierlinski 2003 MNRAS 343 1302, Cadeau Leahy Morsink 2005 ApJ 618 481)

# Light curve generation

- Shape of emission region
- Surface emissivity (energy spectrum, angular distribution of emitted photons)
- Propagation of light to the observer (Schwarzschild metric vs. exact neutron star metric)

# Light propagation in Schwarzschild: Pechenick, Ftaclas & Cohen, 1983

Simple geodesics in Schwarzschild metric:

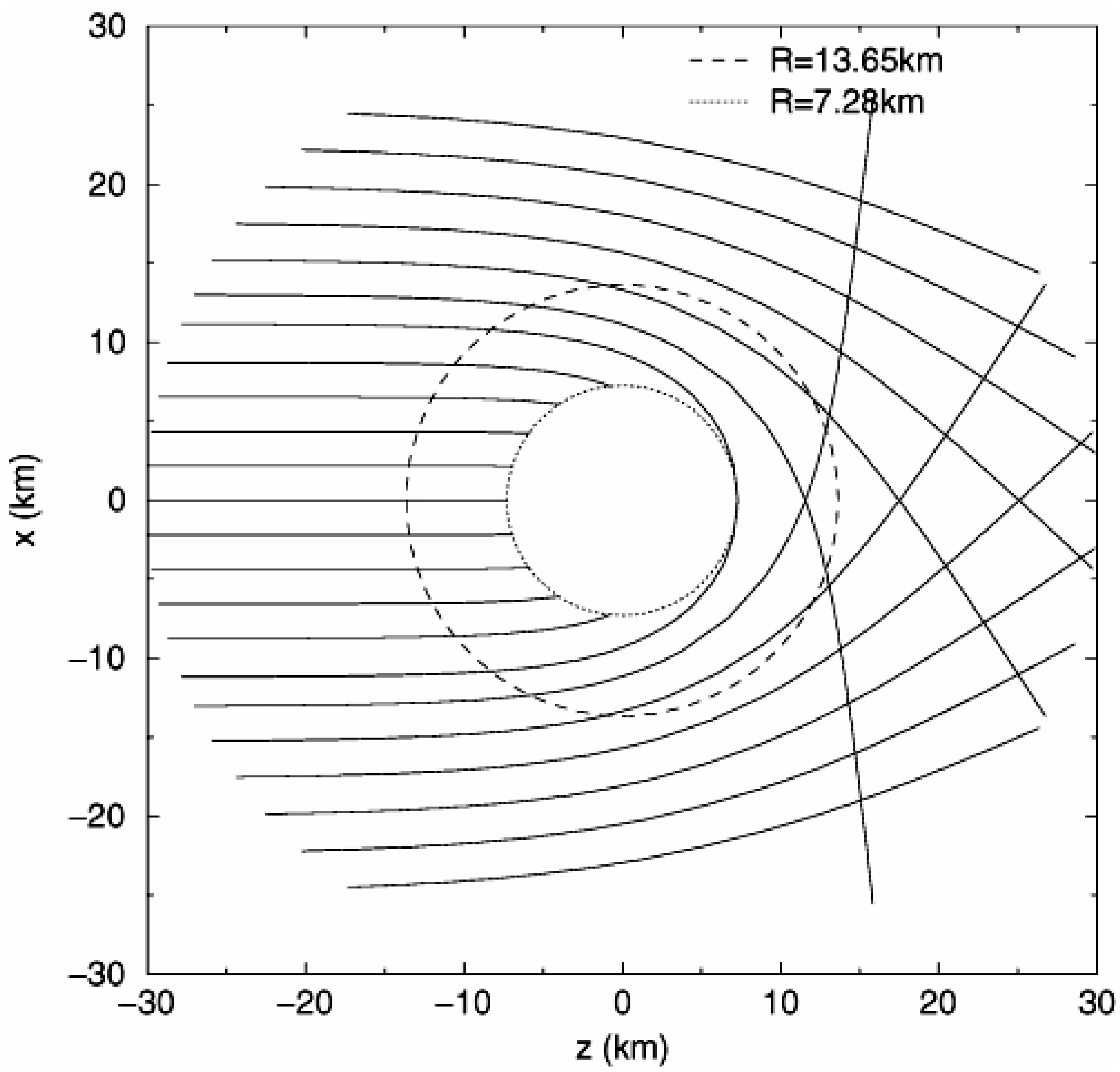
Let us now suppose that the photons are emitted from the surface ($r = R$) of an opaque sphere (the neutron star). The photon described by equation (2.2) obeys (Misner, Thorne, and Wheeler 1973, p. 673)

$$\left(\frac{1}{r^2}\frac{dr}{d\phi}\right)^2 + B^{-2}(r) = \frac{1}{b^2}, \tag{2.7}$$

where

$$B^{-2}(r) = (1 - 2M/r)/r^2. \tag{2.8}$$

# Propagation of light




Ray paths for Schwarzschild to $z \to -\infty$ (observer)

(Rsch=4.15km)

Shadow zone

Part of back surface is visible

Stellar surface is magnified

# Light bending for surface emission I

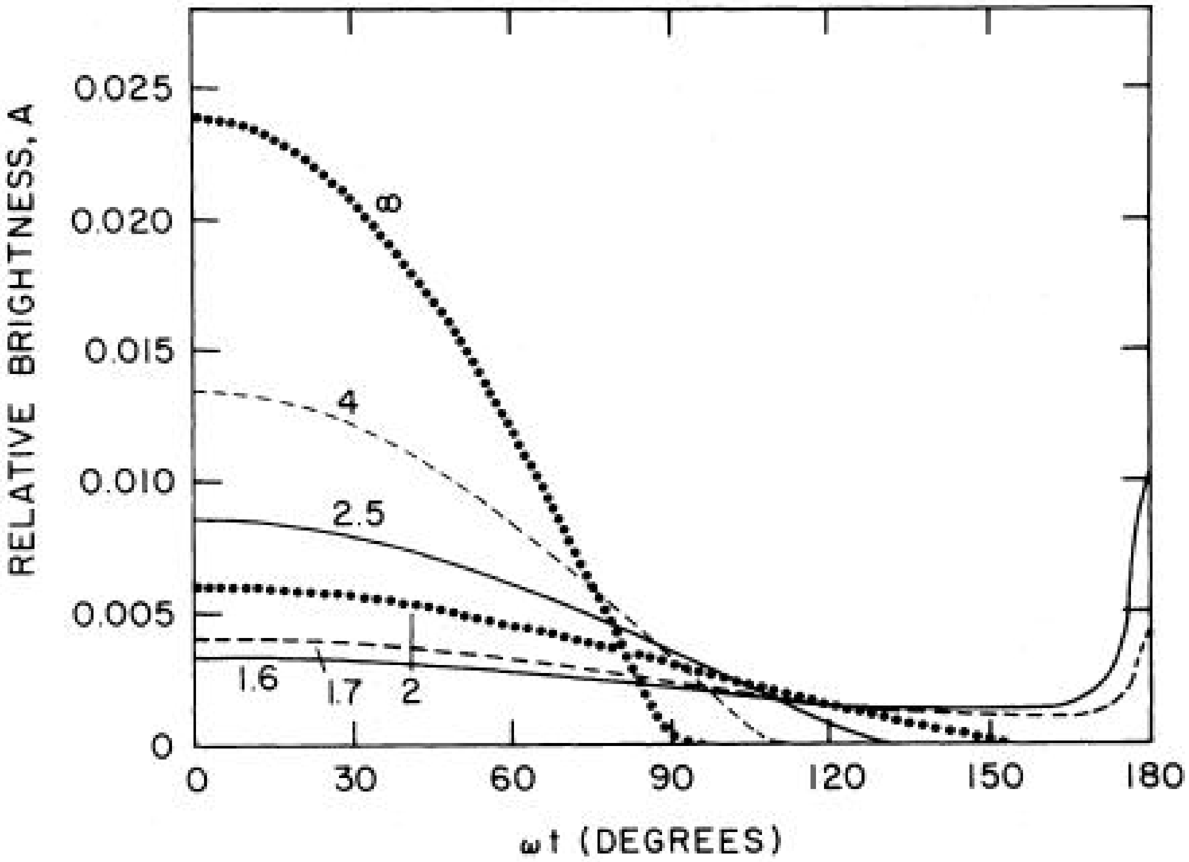
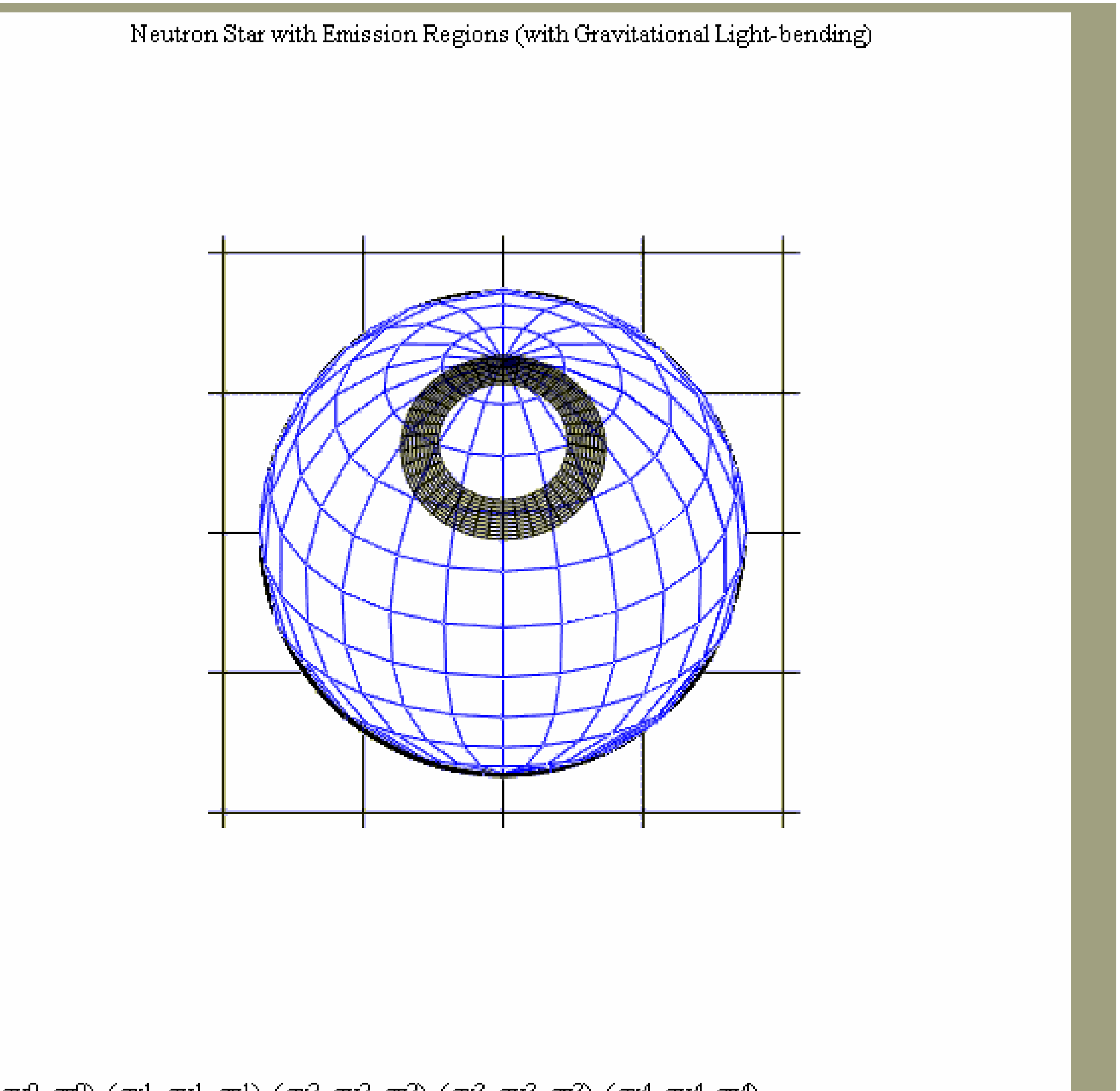



FIG. 4.—Relative brightness $A$ (one polar cap) vs. $\omega t$ for $\alpha = 5°$, $f(\delta) = 1$, $\beta = \gamma = 90°$. Numbers denote values of $R/2M$.

Pechenick, Ftaclas & Cohen, 1983

# Light bending for surface emission II



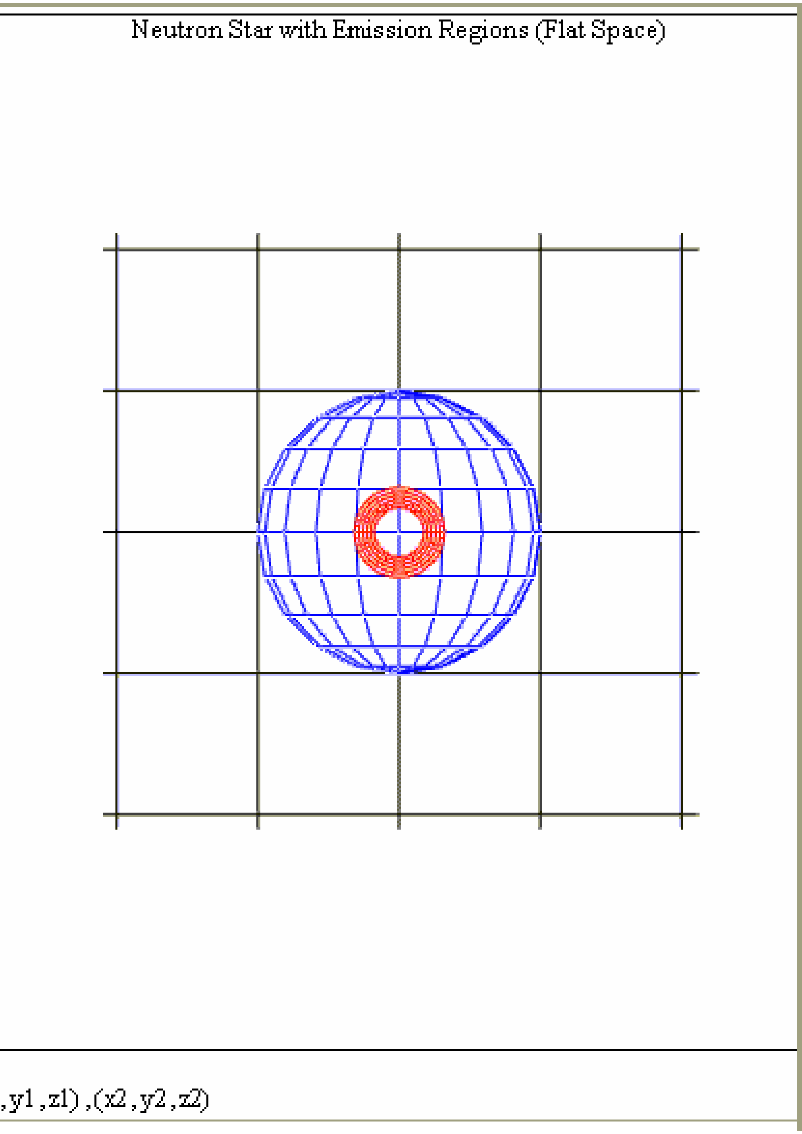

More surface visible

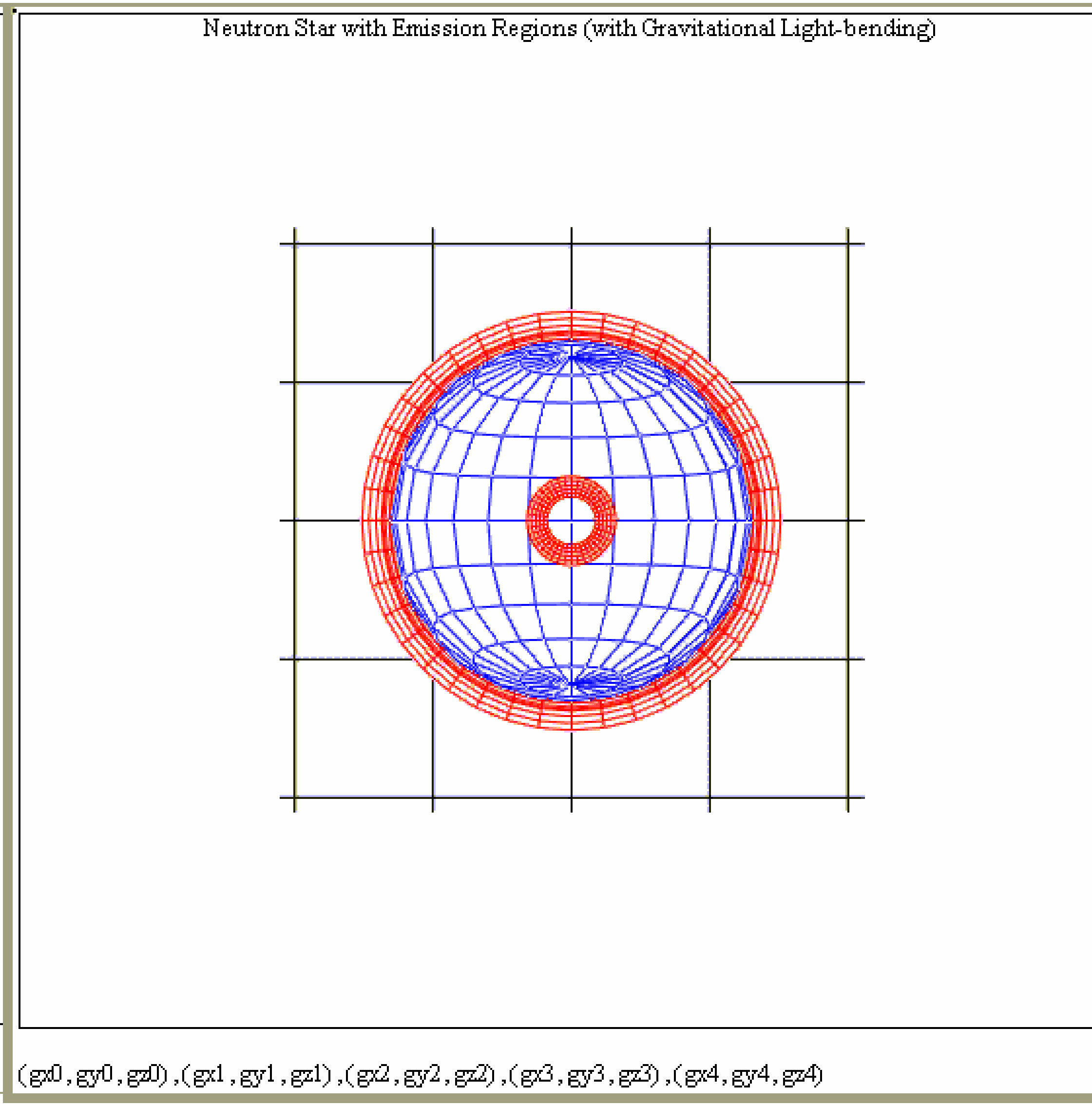
Magnification increases toward limb

Projected area of emission surface goes to zero at limb

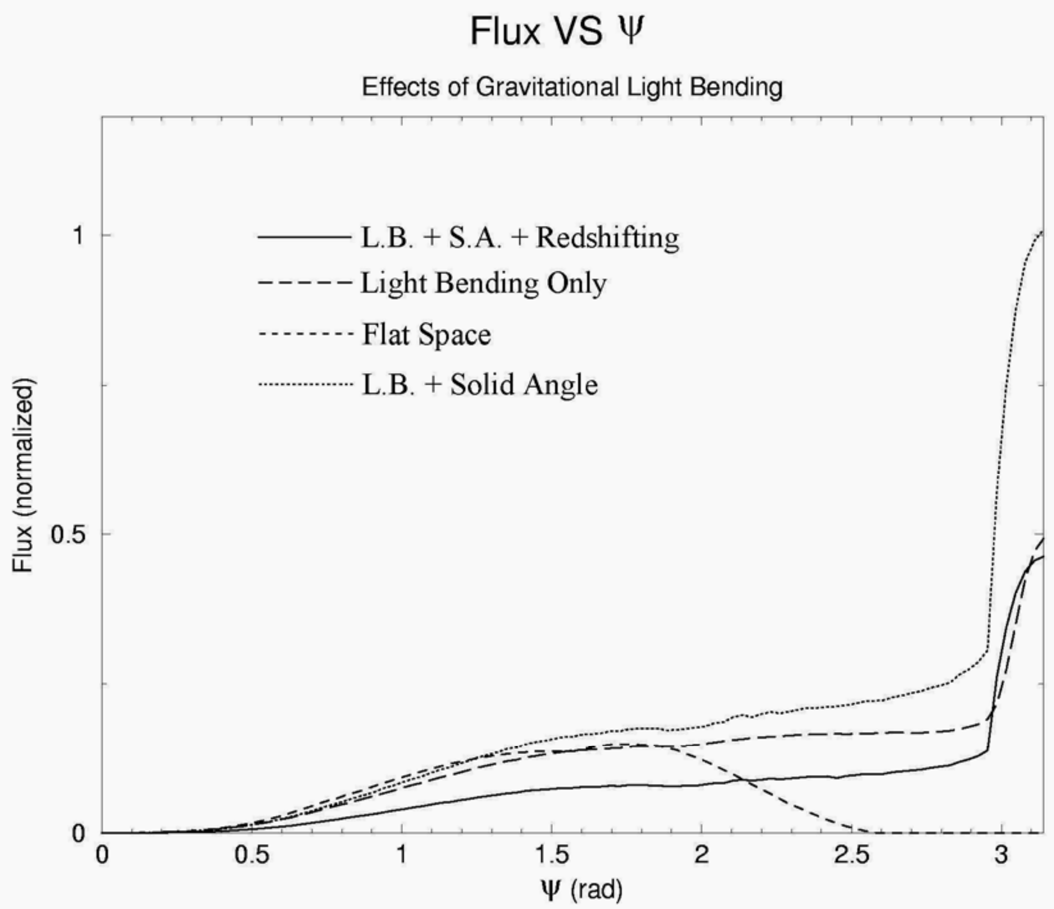
(R/Rsch=2.5)

# Light bending: column I (R/Rs=2.5)



# Light bending: column II




Column exterior emission only

"Spike" of emission due to magnification at limb

Area normal to light ray also max at limb

# What is so different for ms pulsars? I

Ms period means speeds at surface are not small compared to c (v=47000km/s for P=2ms, R=15km)

Special-relativity:
-light aberration effects
-Doppler boosting and de-boosting

Time delays (R/c) are significant compared to the time for the star to rotate

# What is so different for ms pulsars? II

Neutron star is oblate due to fast rotation

Spacetime metric is axisymmetric and must be computed numerically

We use algorithm of Cook, Shapiro and Teukolsky 1994; public domain code of Stergioulas and Friedman 1995

Equations of state from Arnett & Bowers 1977:
EOS A: very soft
EOS L: very stiff

# Light propagation in exact metric:

Axisymmetric spacetime:

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}\left(d\bar{r}^2 + \bar{r}^2 d\theta^2\right) + e^{\gamma-\rho}\bar{r}^2 \sin^2\theta(d\phi - \omega dt)^2$$

Geodesic equations:

$$
\begin{aligned}
\dot{t} &= e^{-(\gamma+\rho)}(1-\omega b) \\
\dot{\phi} &= \omega e^{-(\gamma+\rho)}(1-\omega b) + e^{\rho-\gamma}\frac{b}{\bar{r}^2 \sin^2\theta} \\
\ddot{\bar{r}} &= -\alpha_{,\bar{r}}\left(\dot{\bar{r}}^2 - \bar{r}^2\dot{\theta}^2\right) - 2\alpha_{,\theta}\dot{\bar{r}}\dot{\theta} + \bar{r}\dot{\theta}^2 + \frac{1}{2}e^{-2\alpha}\mathcal{B}_{,\bar{r}} \\
\ddot{\theta} &= \alpha_{,\theta}\left(\frac{\dot{\bar{r}}^2}{\bar{r}^2} - \dot{\theta}^2\right) - 2\left(\alpha_{,\bar{r}} + \frac{1}{\bar{r}}\right)\dot{\bar{r}}\dot{\theta} + \frac{1}{2\bar{r}^2}e^{-2\alpha}\mathcal{B}_{,\theta} \\
\mathcal{B} &= e^{-(\gamma+\rho)}(1-\omega b)^2 - \frac{b^2 e^{\rho-\gamma}}{\bar{r}^2\sin^2\theta}.
\end{aligned}
$$

# Time delays, equatorial rays

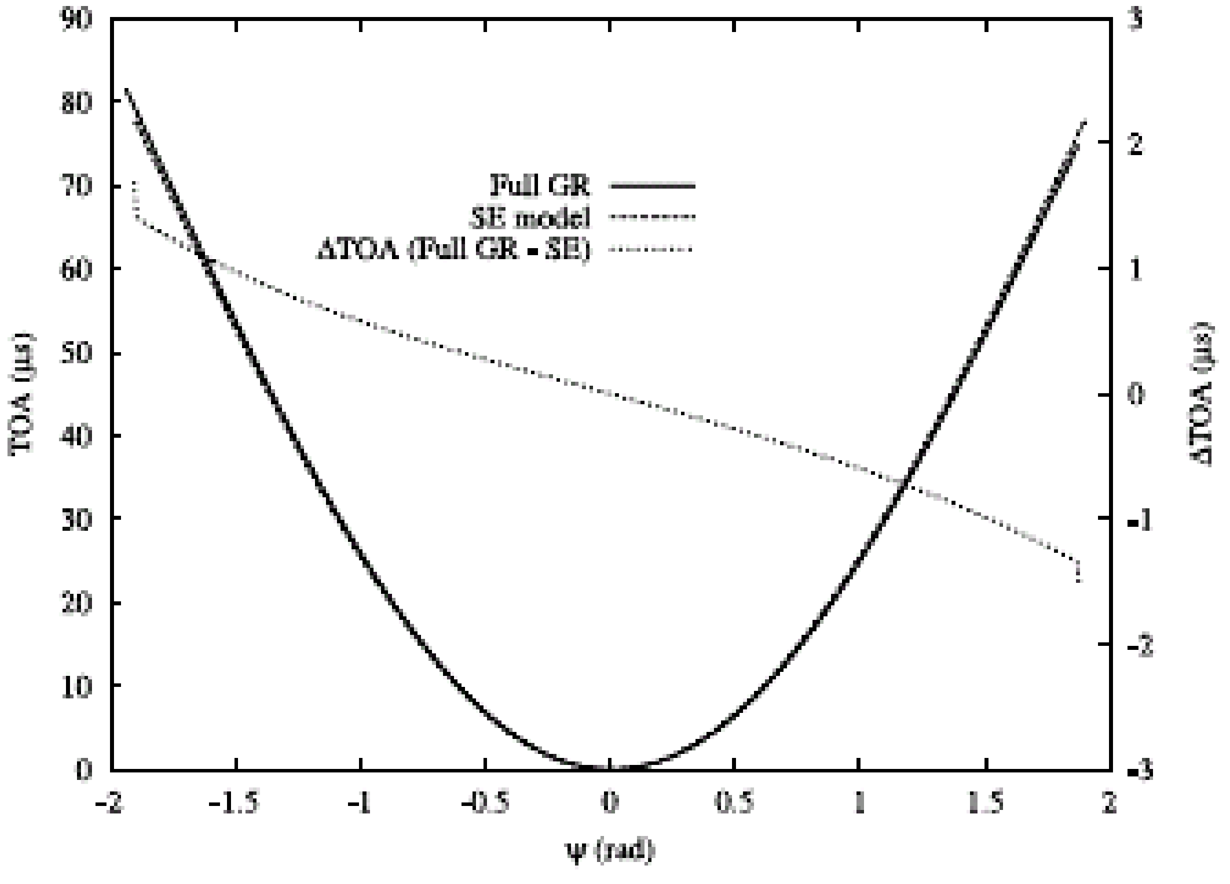



FIG. 3.—TOA as a function of bending angle for model 4, calculated using the full metric and the SE metric.

600 Hz
1.4 Msun
EOS L
$R_{eq}$=16.38km

# Pulse shape, equatorial rays

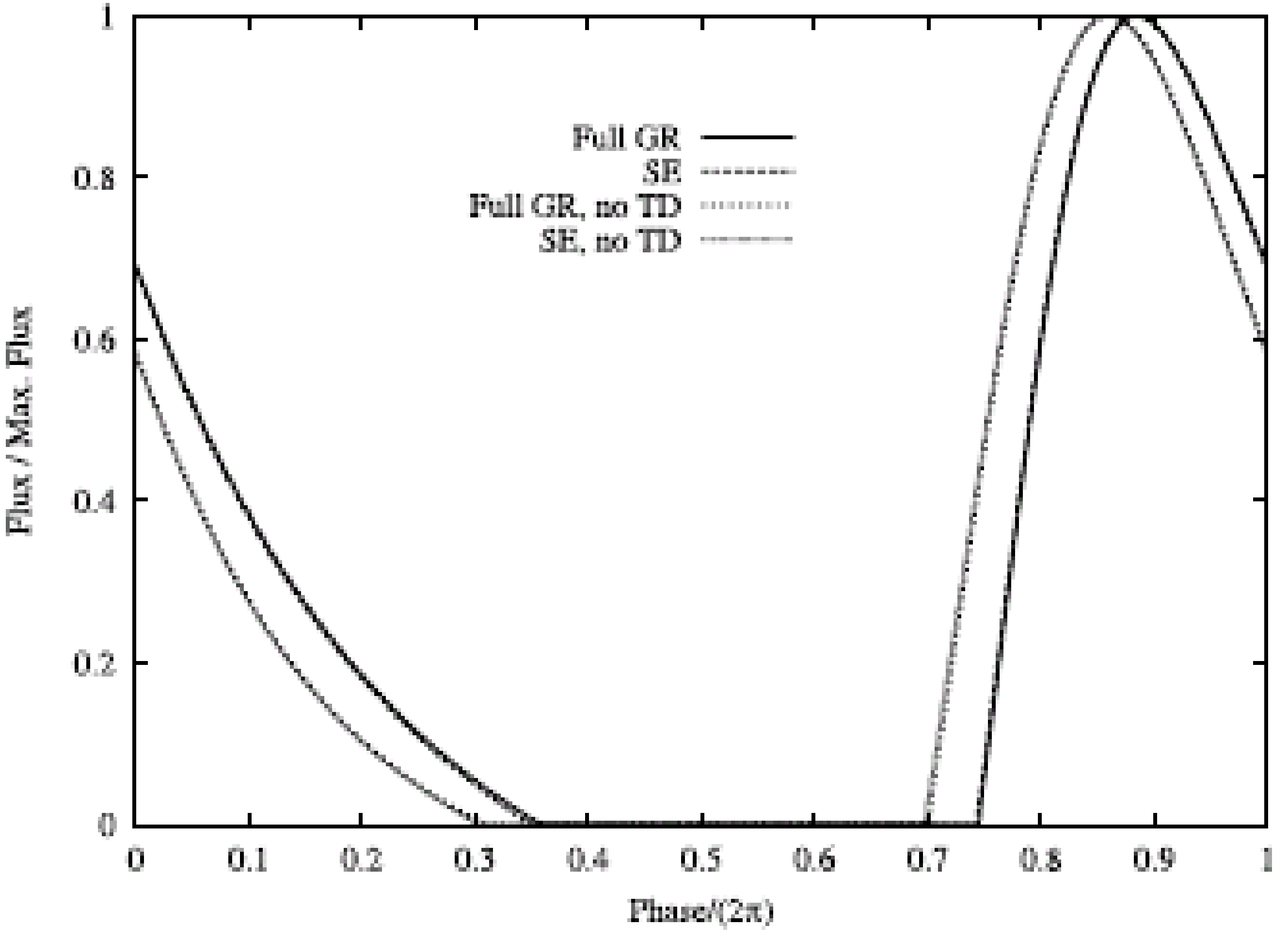
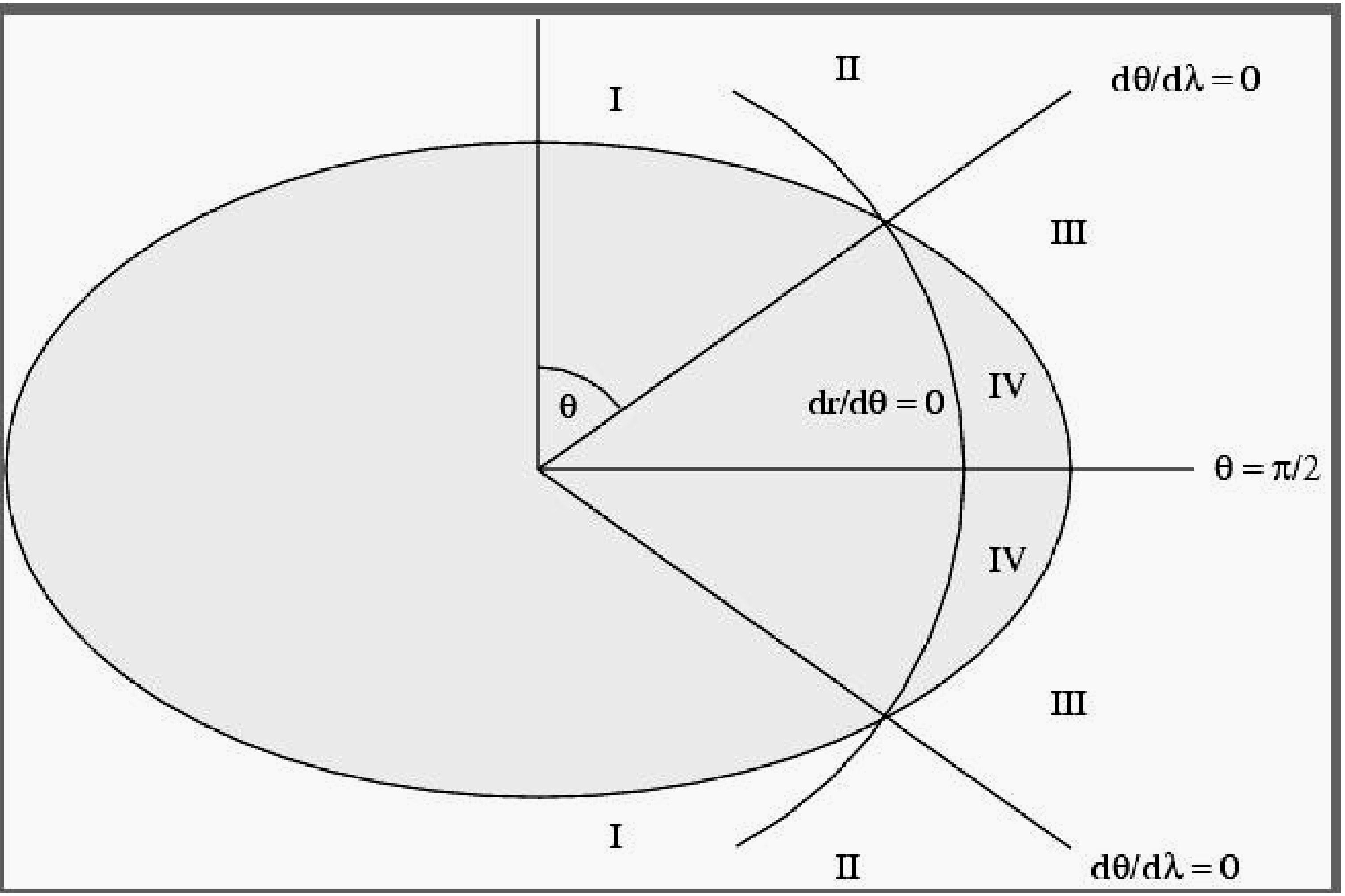



Fig. 5.—Bolometric pulse shapes for model 4 using different calculation methods.

Asymmetric pulse shape due to:

1. Doppler boosting and aberration
2. Time delays

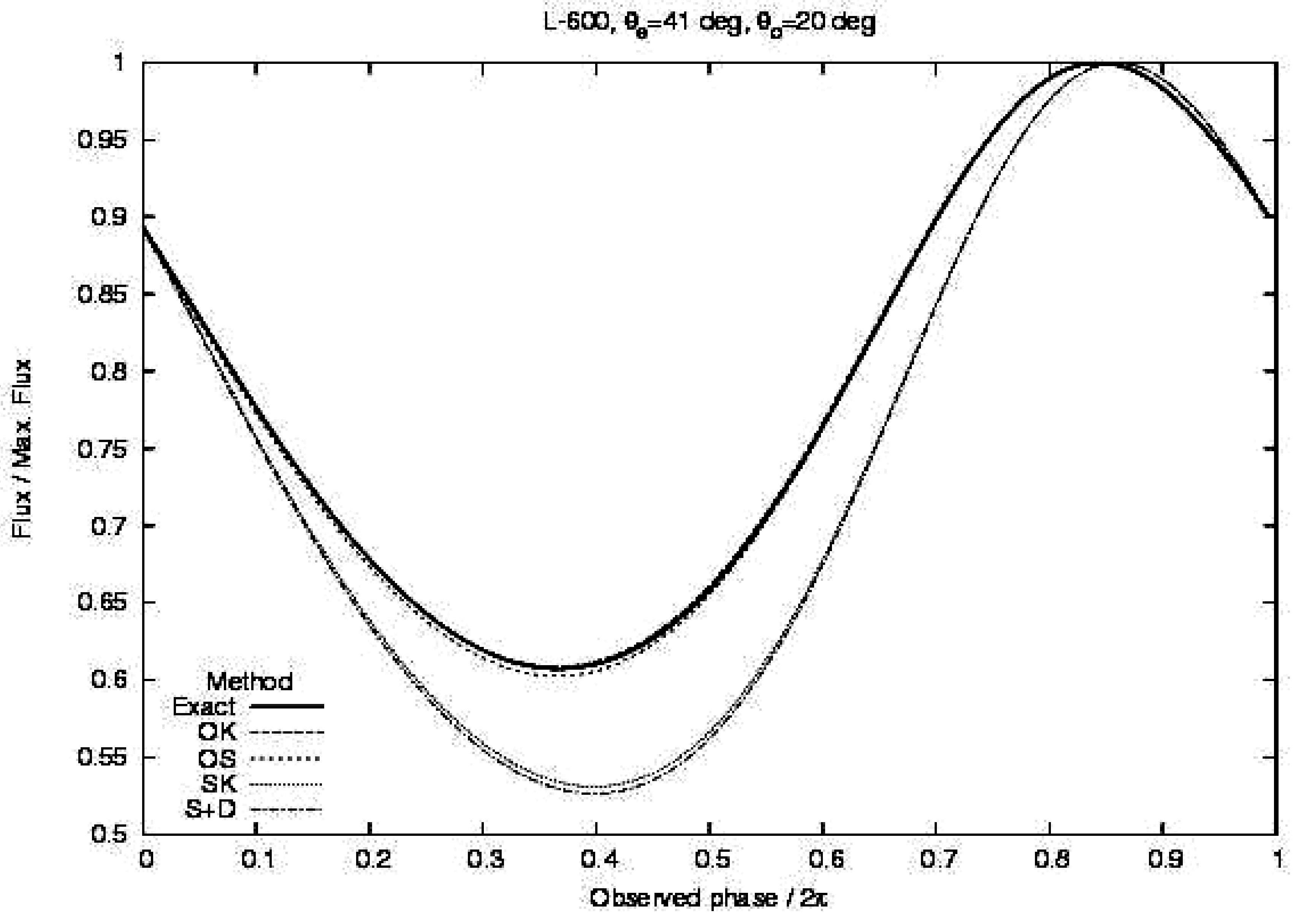
# Oblateness: visibility



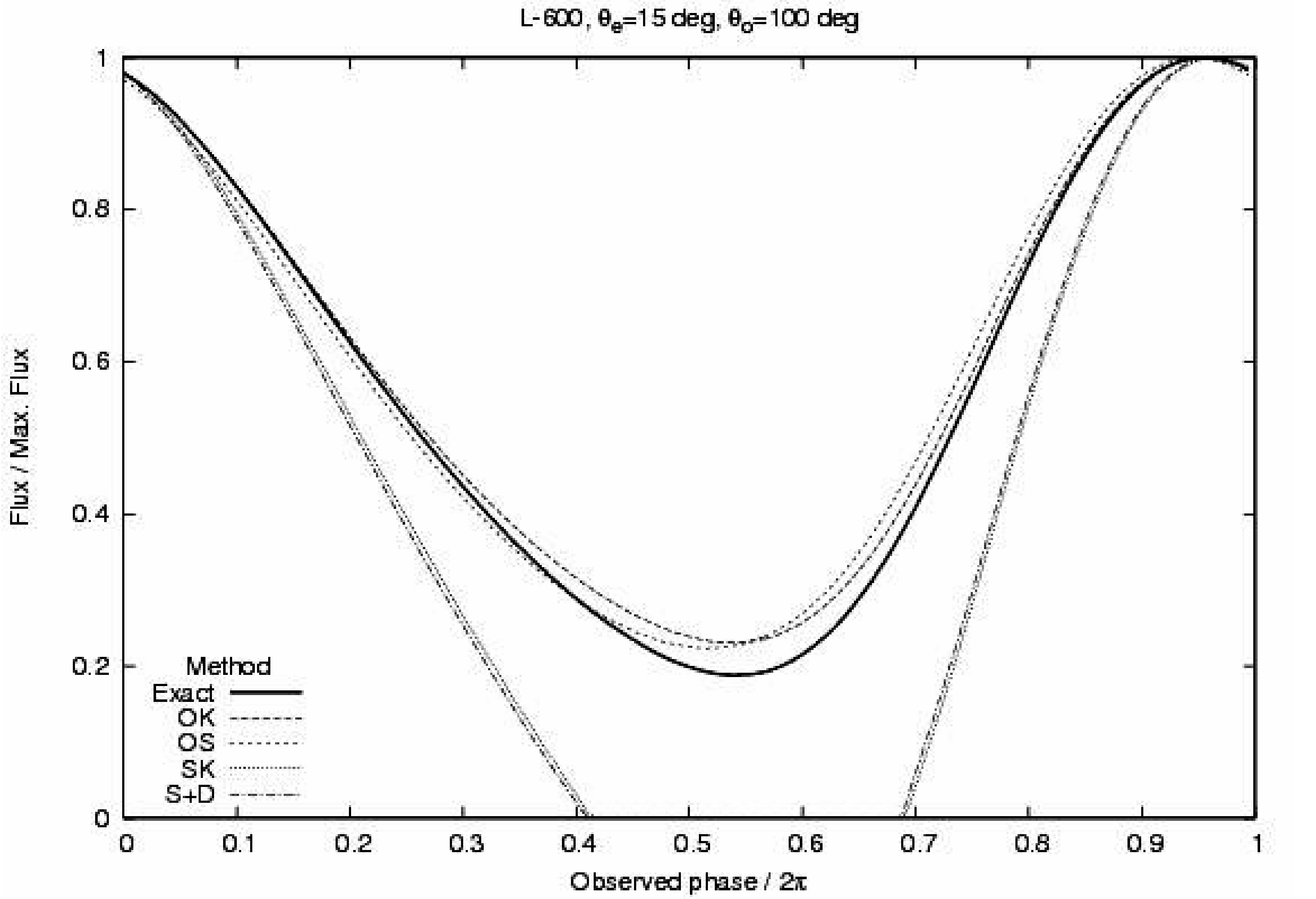

Spherical star: Rays propagate In regions II,III,IV

Oblate star: Rays propagate In regions I,II,III

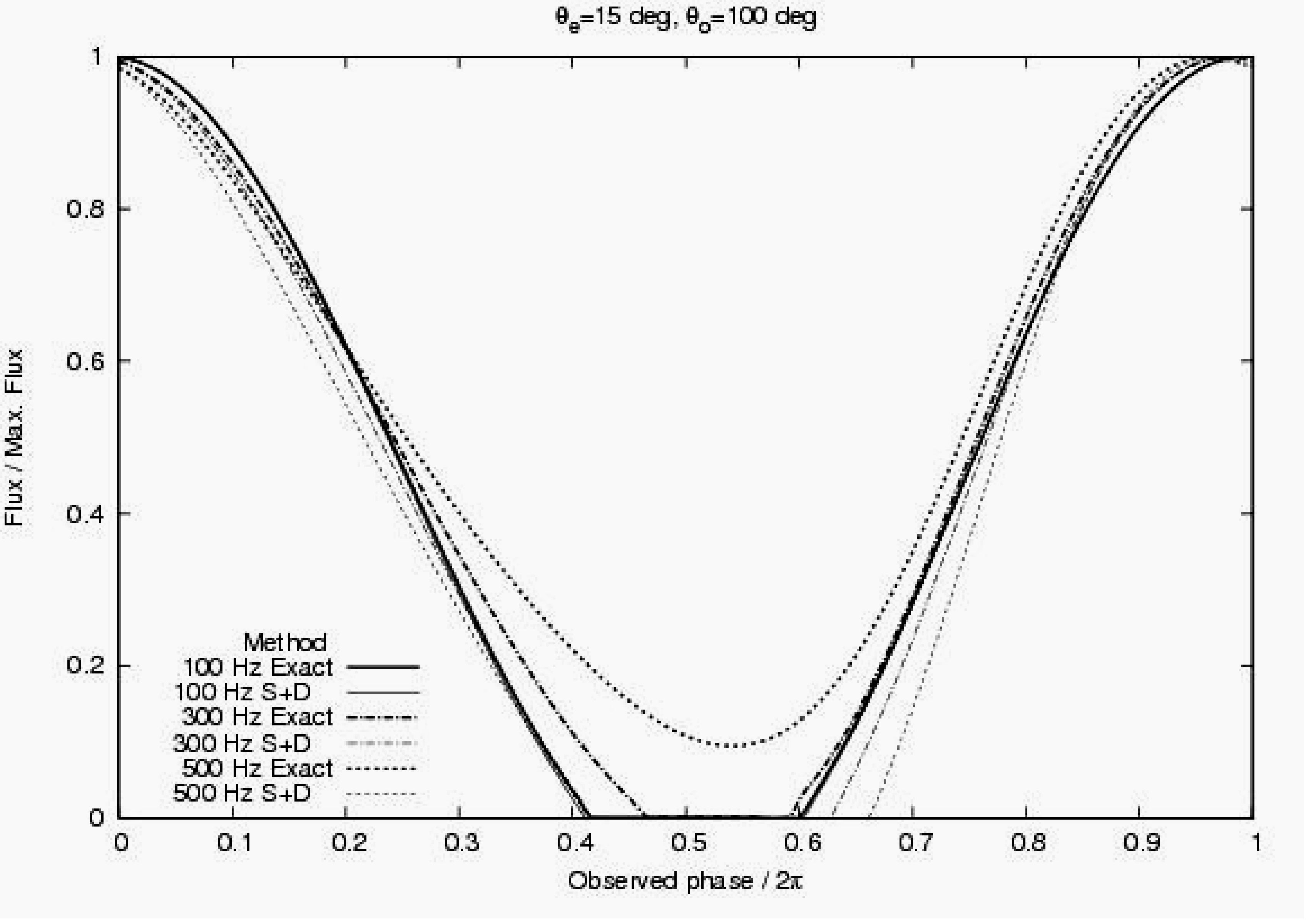
# Different calculation methods I



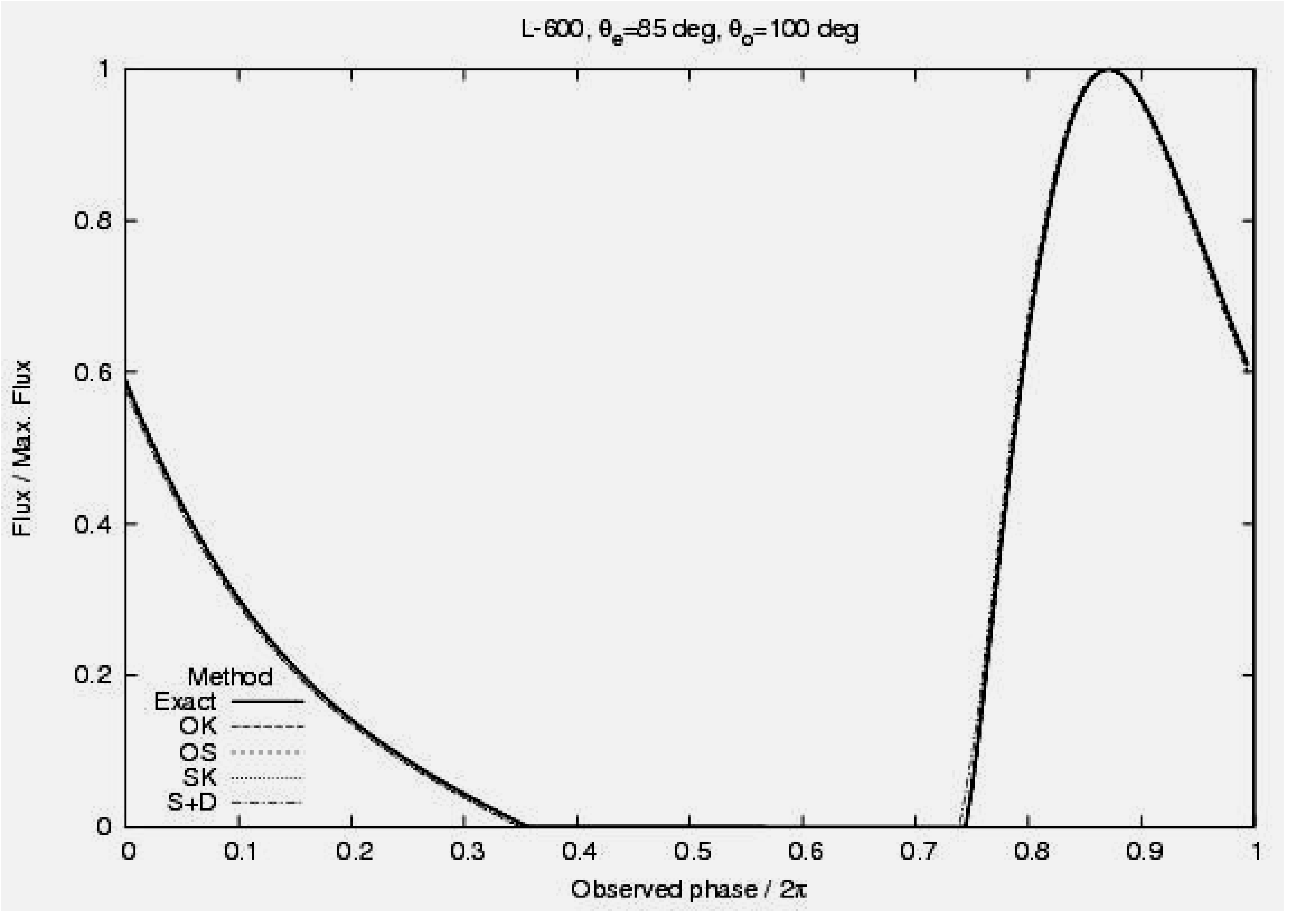
# Different calculation methods II



# Different rotation rates



# Near-equatorial rays



# S+D model fit to exact light curves

Table 1. Comparison of True and Fitted Parameters of Neutron Stars. All neutron stars have a mass of $1.4M_\odot$.

| $\theta_e$ true | $\theta_o$ true | EOS | $\Omega_\star$ (Hz) | $M/M_\odot$ fit | $R(\theta_e)$ (km) true | $R(\theta_e)$ (km) fit | $\theta_e$ (deg) fit | $\theta_o$ (deg) fit | $GM/c^2R(\theta_e)$ true | $GM/c^2R(\theta_e)$ fit | $GM/c^2R(\theta_e)$ unc. | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 41° | 100° | A | 100 | 1.48 | 9.57 | 10.2 | 80.5 | 139.2 | 0.216 | 0.215 | 0.011 | 0.1 |
| | | L | | 1.32 | 14.83 | 13.8 | 80.5 | 133.0 | 0.140 | 0.142 | 0.027 | 0.02 |
| | | A | 300 | 1.49 | 9.58 | 10.0 | 79.8 | 138.0 | 0.216 | 0.220 | 0.005 | 1 |
| | | L | | 1.09 | 14.82 | 11.1 | 67.0 | 95.6 | 0.140 | 0.145 | 0.024 | 0.3 |
| | | A | 400 | 1.45 | 9.58 | 9.55 | 80.8 | 134.9 | 0.216 | 0.225 | 0.006 | 2 |
| | | L | | 1.17 | 14.80 | 11.9 | 58.0 | 96.3 | 0.140 | 0.145 | 0.023 | 0.4 |
| | | A | 500 | 1.51 | 9.59 | 9.89 | 80.2 | 136.9 | 0.216 | 0.225 | 0.005 | 3 |
| | | L | | 1.29 | 14.78 | 12.7 | 52.7 | 98.1 | 0.140 | 0.15 | 0.02 | 0.8 |
| | | A | 600 | 1.58 | 9.60 | 10.2 | 41.9 | 102.2 | 0.215 | 0.230 | 0.007 | 4 |
| | | L | | 1.30 | 14.74 | 12.0 | 57.9 | 97.5 | 0.140 | 0.160 | 0.015 | 2 |

# S+D fits, cont'd

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 35° | 100° | **A** | 100 | 1.48 | 9.57 | 10.4 | 87.4 | 110.8 | 0.216 | 0.210 | 0.008 | 1 |
| | | L | | 1.45 | 14.86 | 15.3 | 84.0 | 103.7 | 0.139 | 0.140 | 0.027 | 0.05 |
| | | **A** | 200 | 1.46 | 9.59 | 10.1 | 84.4 | 103.9 | 0.216 | 0.215 | 0.006 | 2 |
| | | L | | 1.43 | 14.95 | 15.6 | 86.7 | 107.6 | 0.138 | 0.135 | 0.029 | 0.4 |
| | | **A** | 300 | 1.48 | 9.62 | 10.2 | 83.1 | 103.5 | 0.215 | 0.215 | 0.025 | 4 |
| | | L | | 1.70 | 15.10 | 17.9 | 80.0 | 123.3 | 0.137 | 0.140 | 0.015 | 0.4 |
| | | **A** | 400 | 1.49 | 9.66 | 10.3 | 77.2 | 99.7 | 0.214 | 0.215 | 0.003 | 7 |
| | | L | | 1.40 | 15.35 | 16.0 | 85.0 | 105.5 | 0.135 | 0.130 | 0.009 | 4 |
| | | **A** | 500 | 1.68 | 9.71 | 11.3 | 67.7 | 111.0 | 0.213 | 0.220 | 0.004 | 5 |
| | | L | | 1.51 | 15.73 | 17.9 | 61.7 | 97.7 | 0.131 | 0.125 | 0.009 | 6 |
| | | **A** | 600 | 1.62 | 9.78 | 11.1 | 72.5 | 113.6 | 0.211 | 0.215 | 0.003 | 0.1 |
| | | L | | 1.40 | 16.35 | 17.3 | 77.8 | 102.6 | 0.127 | 0.120 | 0.007 | 0.2 |

# Polar cap model for SAX J1808

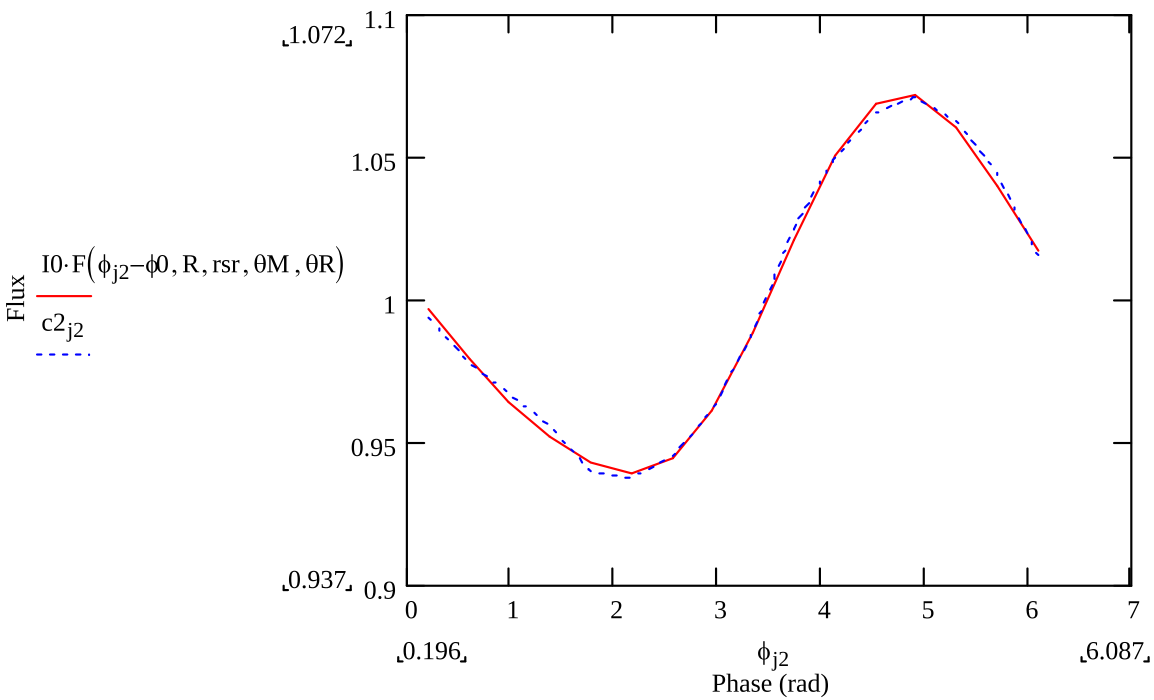
Known pulse period (401 Hz)

Pulse shape is nearly symmetric

Constraints on model parameters are loose

Rs/R=.65, R=8.1 km
M=1.6 Msun if no time delays included

Rs/R=.59, R=6.3 km
M=1.3 Msun if time delays included



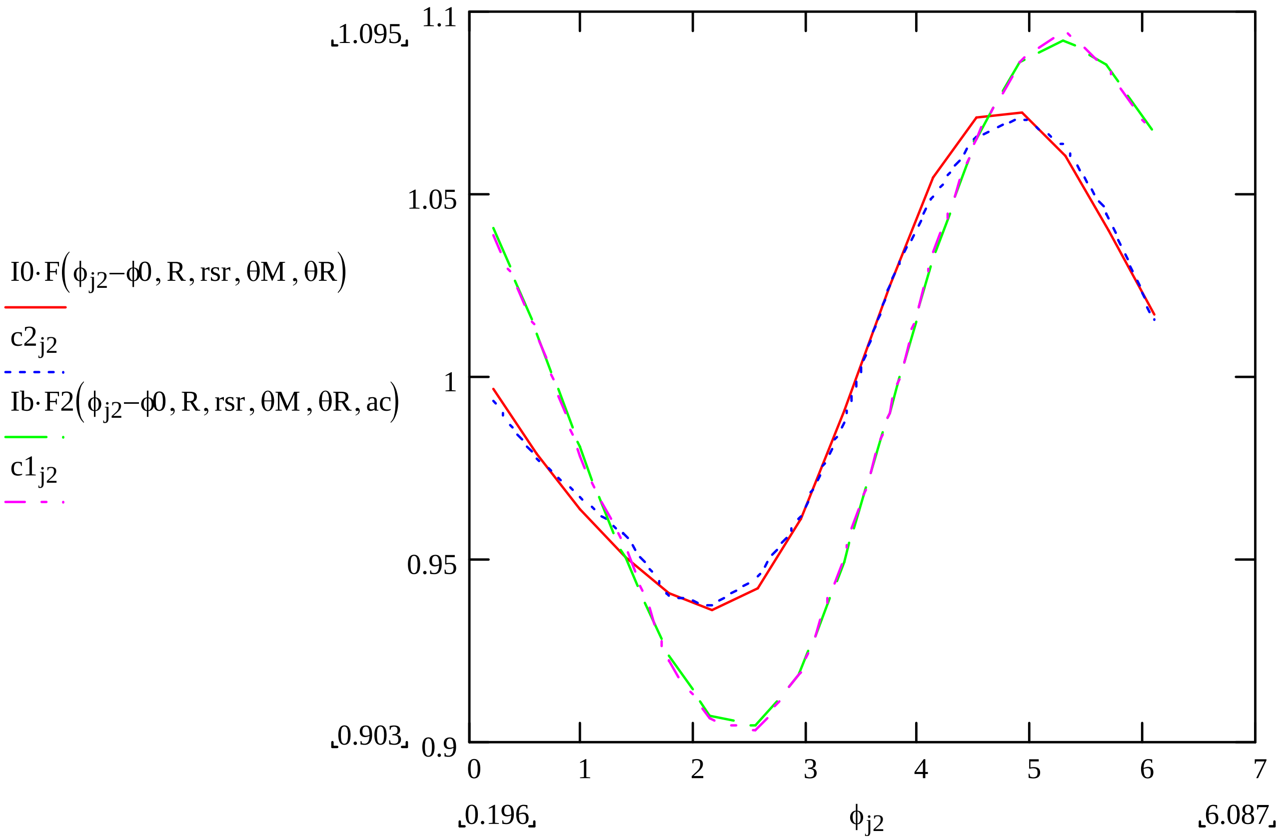

12-18 keV

# Two energy bands fit

Much better constrained

Rs/R=.65, R=7.2 km M=1.6 Msun if no time delays included

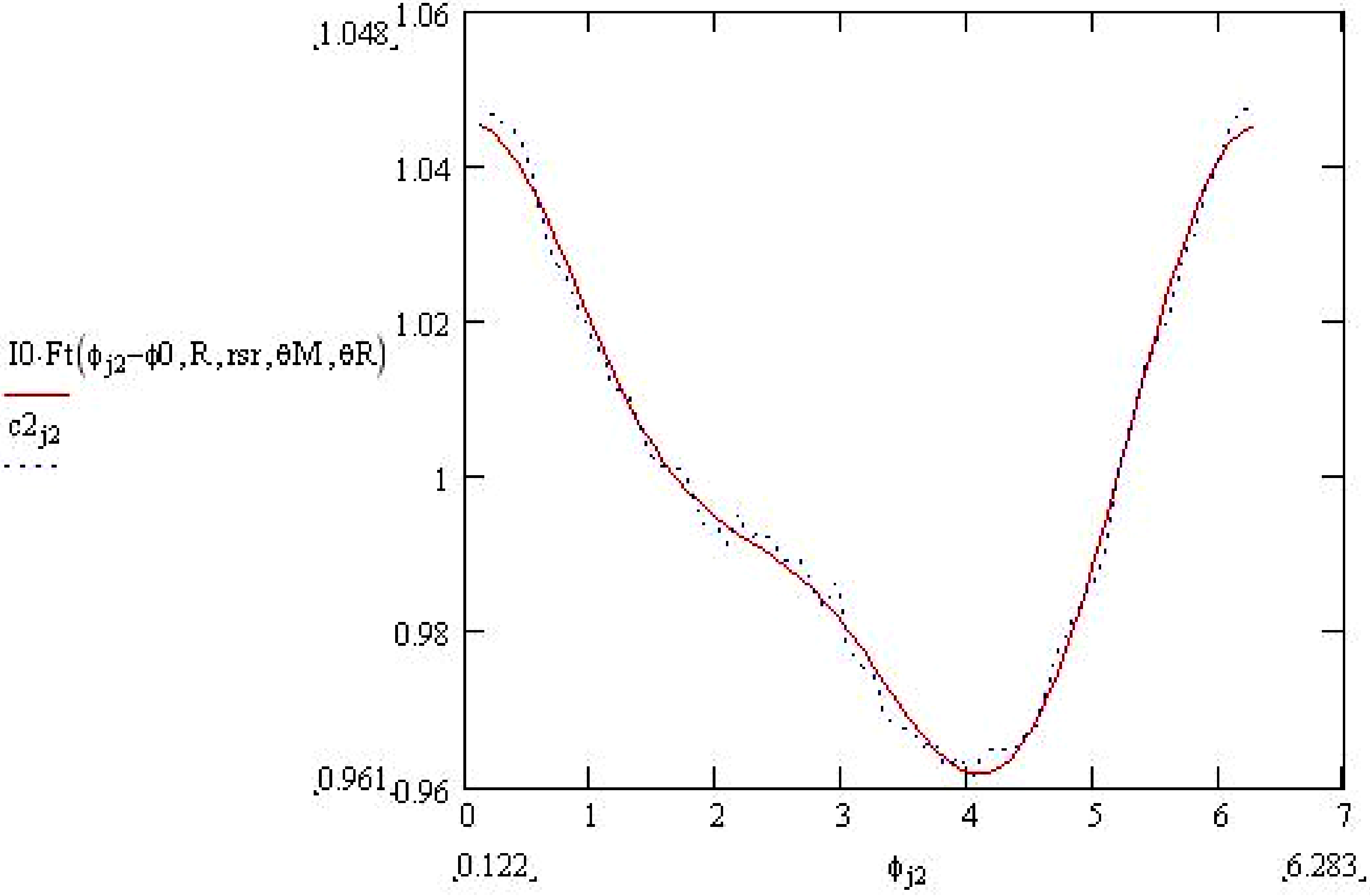
Rs/R=.59, R=8.3 km M=1.65 Msun if time delays included




3-4 keV: blackbody + Compton
12-18 keV: Compton

# New data Oct.2002

Best fit:
R=4.8km
M=0.69Msun

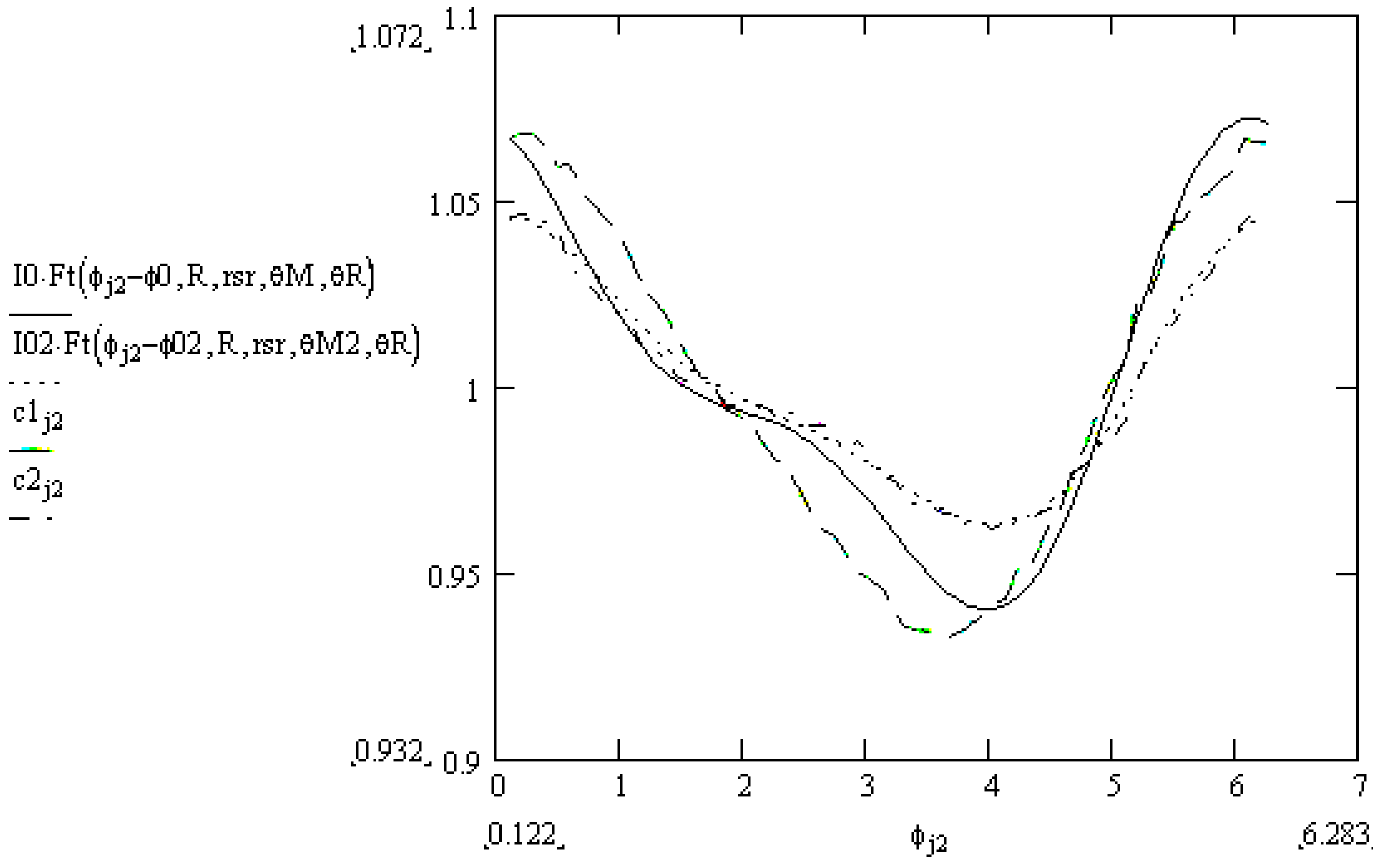


# Joint fit: 1998 & 2002

R=6.0 km
M=0.94 Msun
R/Rs=2.15

Rotation axis
68deg
Hot spot:
  17deg(Apr98)
  12deg(Oct02)

Oblateness not
yet included



# Fit results II

R=6.0 km, M=0.94 Msun: Quark star??

Not yet.

Compare to fitting to exact calc for small angles: Mfit & Rfit can be too large or too small by up to 50% due to missing oblateness

More realistic guess: R<10km, M<1.5Msun

Coming soon: approx. model with oblateness.

# Summary

- New aspects of exact calculations: ray tracing in full metric, time delays, oblate neutron star
- Oblateness and time delays are largest effects
- Exact calc. too slow for multi-parameter fitting to data
- Approx. calc: currently incorporating oblateness
- Fitting results sensitive to including all effects
- Goal: obtain M/R, M, R and emission region parameters for ms pulsars