Stable MHD Equilibria in Young Neutron Stars: Magnetars or Radio Pulsars

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"Initial" Magnetic Field Configuration?

Tightly connected phases

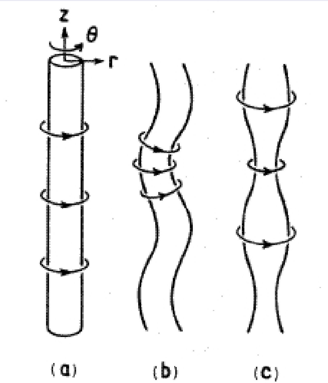
- state at the end of each phase
  $=$ initial conditions of the next
- SN–Progenitor : $B_\varphi = 5 \cdot 10^9$G, $B_r = 10^6$G,
- Core–Collapse : conservation of flux, angular momentum
- Proto–Neutron Star : $\sim$ 20 s convection $\Longrightarrow$ dynamos
- Young Neutron Star : thermoelectric instabilities,
  MHD–instabilities

Birth of Neutron Stars in Supernovae

- NSs born with $B \sim 10^{15}$G
- few of them seen as magnetars (AXP, SGR)
- $E_{\mathrm{mag}} \sim E_{\mathrm{rot}}, E_{\mathrm{therm}}$, released in GRBs
- How does the establishment of $\boldsymbol{B}_{\mathrm{Magnetar}}$ proceed?

Stability of $\boldsymbol{B}(\boldsymbol{r}, t)$ in a Conducting Fluid

Pinch type instability



equilibrium | kink $(m=1)$ | varicose $(m=0)$

Early Work

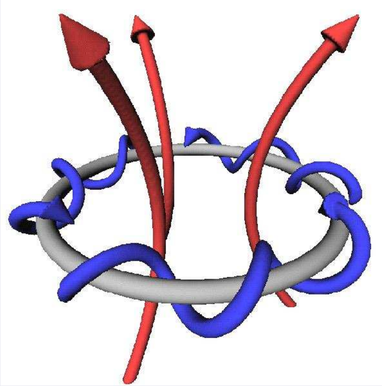
- Prendergast '56: $\boldsymbol{B}^{tor} \sim \boldsymbol{B}^{pol}$
- Tayler et al. '73: $\boldsymbol{B}^{tor}$ always unstable with respect to $m = 1$ perturbations, $p \sim 1/\tau_{\mathrm{A}}$
- Pitts & Tayler '85: $\boldsymbol{B}^{tor}$ + uniform $\boldsymbol{B}^{pol}$: rotation may stabilize AND destabilize $\Longrightarrow$ "rotation is unlikely to stabilize general $\boldsymbol{B}$ "

Stability of $\boldsymbol{B}(\boldsymbol{r}, t)$ in a Conducting Fluid

Recent Work

- Braithwaite & Nordlund, $\sim$ & Spruit '05: $\boldsymbol{B}^{tor} \sim \boldsymbol{B}^{pol}$
  torus–like field stable
  if concentrated in inner core
- Braithwaite & Spruit '05:
  application to non–rotating Ap–stars
- application to non–rotating magnetars
  but: $B_s > 10^{14}$ G questionable

"Twisted torus" equilibrium



Effect of rotation ($P < \tau_A$!) on stability of poloidal/toroidal fields not yet considered.

Stable MHD–Equilibria in Young Neutron Stars

What Does "Young" Mean?

- Immediately after PNS–phase
- Whole NS liquid, $T \gtrsim 10^{10}$
- $\tau_{\mathrm{cryst}} \sim \tau_{\mathrm{SF}} \sim 1000$ sec,

... let's take Thompson & Duncan '93 seriously:

- Rapid rotation: $P = 0.6 \ldots 60$ ms
- Highly magnetized: $B \gtrsim 10^{15}$G

Model Assumptions

- Rigid rotation: Ott et al. '05: viscous ($\boldsymbol{B}$) dissipation in PNS: $\Omega(r) \Longrightarrow \Omega_{\mathrm{rigid}}$
- Incompressibility: $v_{\mathrm{flow}} \ll c_{\mathrm{s}} = 8 \cdot 10^{8} \cdots \lesssim c$, $\tau_{\mathrm{s}} \ll \tau_{\mathrm{flow}}$
- Constant density (?): flow concentrated within the core $\Longrightarrow$ anelastic approximation

Timescales

- Rotation: $P \sim$ millisecond
- Ohmic decay:
  $\sigma \approx 10^{24}\mathrm{s}^{-1}, \quad R \approx 10^{6}\mathrm{cm}$
  $\Longrightarrow \tau_{\mathrm{Ohm}} \sim 1.4 \cdot 10^{17}\mathrm{s} \approx 4.4 \cdot 10^{8}\mathrm{yrs}$ $\Leftrightarrow$ problem of extremely large Rm
- Viscous dissipation:
  $\tau_{\mathrm{visc}} = R^2/\nu \lesssim \tau_{\mathrm{Ohm}}$
- MHD waves:
  $\tau_{\mathrm{A}} = \sqrt{4\pi\rho}R/B \approx 0.05\mathrm{s}$
  $(\rho = 2 \cdot 10^{14}\mathrm{g\ cm}^{-3})$
  $\Rightarrow \tau_{\mathrm{A}} \sim 10^{-25}\tau_{\mathrm{Ohm,\ visc}}$: will help to get reliable results for stability!

Parameters

- rotation period
  $6\ \mathrm{ms} \leq P \leq 0.6\mathrm{s}$
- inclination angle $\alpha(\boldsymbol{B}, \boldsymbol{\Omega})$
- magnetic Prandtl number
  $Pm = 0.1, 1, 10$
  $(Pm \propto T^{-4})$
- magnetic field strength:
  $\tau_{\mathrm{A}}/\tau_{\mathrm{Ohm}} \sim 10^{-3}$

Mathematical model

Equations (dimensionless)

induction equation + “BC”

$$\frac{\partial \boldsymbol{B}}{\partial t} = \Delta\, \boldsymbol{B} + \mathrm{curl}(\boldsymbol{u} \times \boldsymbol{B}) \quad \text{in } V'$$

$$\mathrm{rot}\, \boldsymbol{B} = \boldsymbol{0} \qquad \text{in } V \setminus V'$$

$$\mathrm{div}\, \boldsymbol{B} = 0 \qquad \text{in } V$$

$$[\boldsymbol{B}] = \boldsymbol{0} \qquad \text{on } \partial V'$$

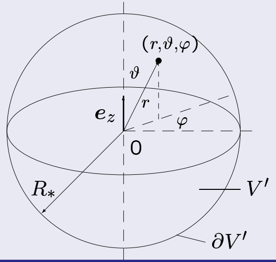
momentum equation + BC

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\nabla p - (\boldsymbol{u}\nabla)\boldsymbol{u} + Pm\,\Delta\, \boldsymbol{u} - 2\Omega \boldsymbol{e}_z \times \boldsymbol{u} + \mathrm{rot}\, \boldsymbol{B} \times \boldsymbol{B}$$

$$\mathrm{div}\, \boldsymbol{u} = 0 \qquad \text{in } V'$$

$$\boldsymbol{u} \cdot \boldsymbol{r} = \left(\underset{\sim}{D}(\boldsymbol{u}) \cdot \boldsymbol{r}\right)_{\vartheta,\varphi} = 0 \quad \text{on } \partial V$$

Geometry



Parameters

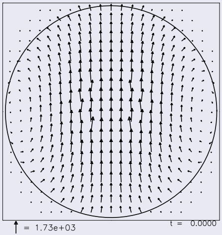
$Pm = \nu/\eta$ – magnetic Prandtl number

Ω – normalized angular velocity

$q_P = P/\tau_{\mathrm{A}}$ – relative rotation period

Dipolar Background Fields

Dipolar equilibrium

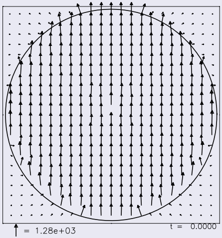


$$\boldsymbol{B} \propto \nabla \times (\boldsymbol{r} \times \nabla \left(\frac{3r^3 - 5r}{4} \cos\vartheta \right))$$

$$\boldsymbol{B} \propto -\nabla \times (\boldsymbol{r} \times \nabla \left(\frac{\cos\vartheta}{2r^2} \right))$$

$$[\boldsymbol{B}] = 0$$

Internal uniform field



$$\boldsymbol{B} \propto \quad \boldsymbol{e}_z$$

$$\boldsymbol{B} \propto -\nabla \times (\boldsymbol{r} \times \nabla \left(\frac{\cos\vartheta}{2r^2} \right))$$

$$[B_r] = 0$$

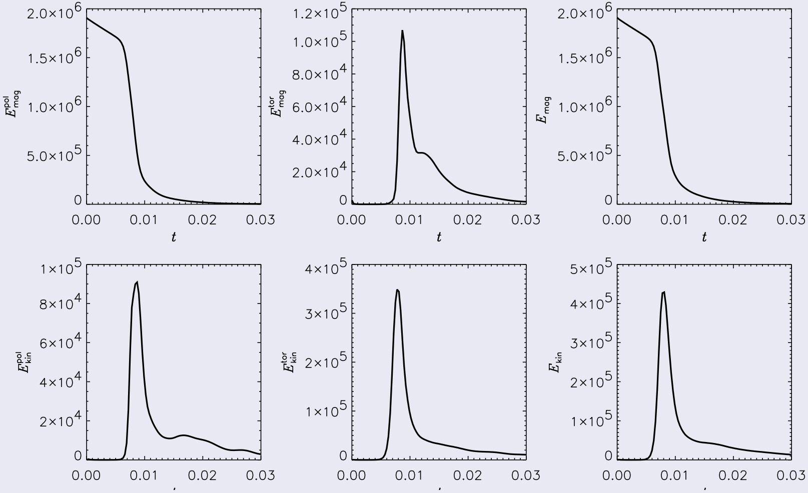
Results for dipolar magnetostatic equilibrium:

for $\tau = \tau_{\text{decay}}$:

α	P/τ_A	$E_{\text{mag}}/E_{\text{mag}}^{\text{Ohm}}$	$E_{\text{kin}}/E_{\text{mag}}$	Π_{mag}	M_{mag}	symmetry
0	∞	0.00018	4.67	-0.06	0.4	mixed
	12	0.00035	6.6	-0.5	0.2	mixed
	1.2	0.076	0.00065	-0.99	0.06	mixed
	0.12	0.98	0.00003	-1	0	**A0**
45	1.2	0.075	0.001	-0.46	0.27	mixed
	0.12	0.82	0.00005	-0.24	0.38	mixed
90	1.2	0.043	0.003	0.88	0.81	mixed
	0.12	0.14	0.00014	1	1	S1
	0.012	0.98	0.0013	1	1	**S1**

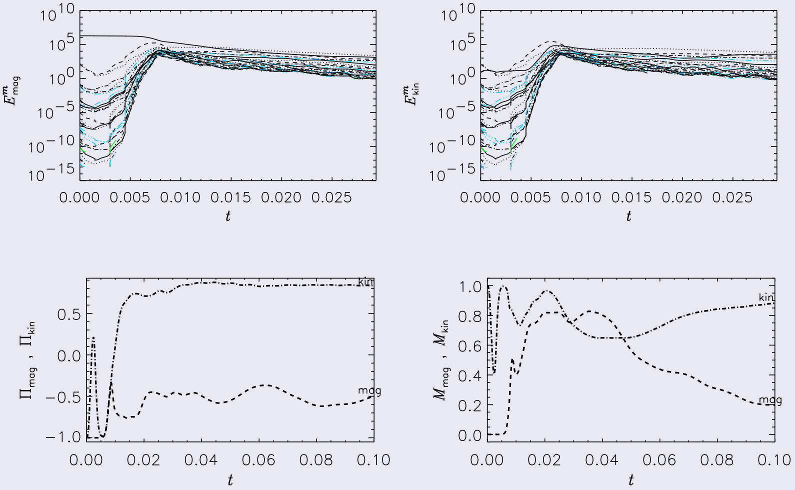
Unstable decay: $P = 0.6$ s, $\alpha = 0$

Energies:



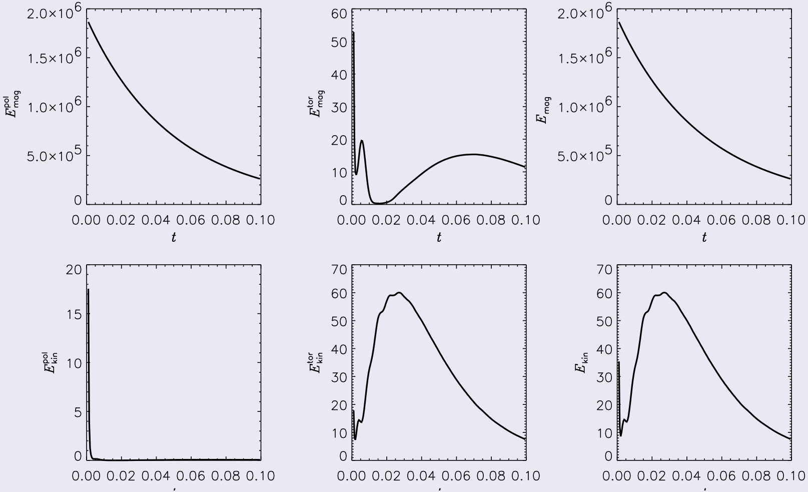
Unstable decay: $P = 0.6$ s, $\alpha = 0$

Spectra, parities, non-axisymmetries:



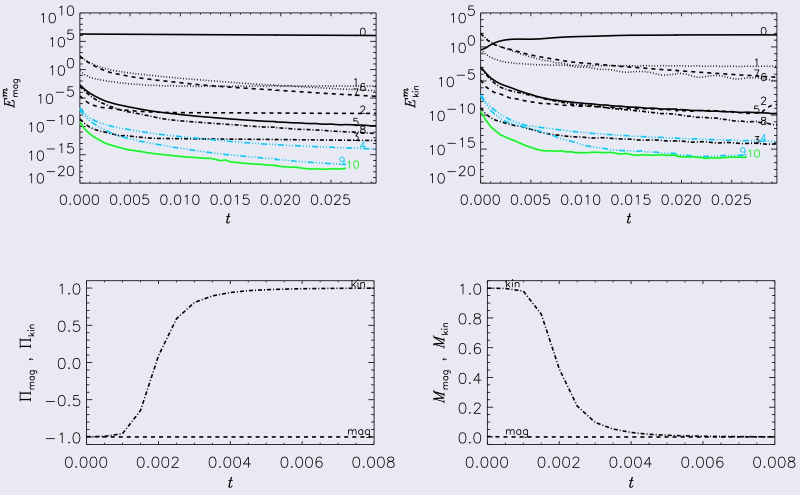
Rotation-stabilized decay: $P = 6$ ms, $\alpha = 0$

Energies:



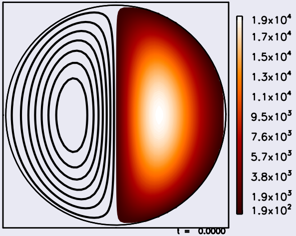
Rotation-stabilized decay: $P = 6$ ms, $\alpha = 0$

Spectra, parities, non-axisymmetries:



Toroidal Background Field

Toroidal initial field



$$\boldsymbol{B} = \frac{s}{s_0} e^{-\left(\frac{s}{s_0}\right)^2} (1 - r^2) \boldsymbol{e}_\varphi$$

$$\boldsymbol{B} = \boldsymbol{0} \quad \text{outside}, \quad s_0 = 0.7$$

↗ Braithwaite 2005

- similar, but
  initial field = exact
  equilibrium
- cylindrical geometry
- rotation
  the only stabilizing agent

Toroidal Background Field: Results

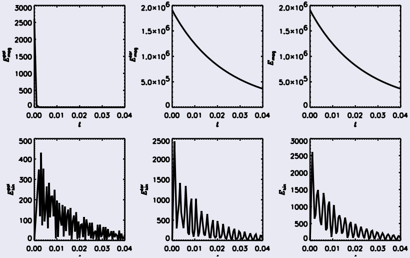
for $\tau = \tau_{\text{decay}}$:

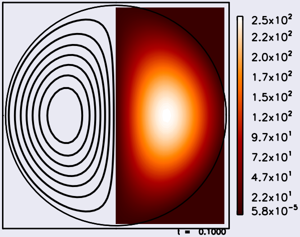
Pm	P/τ_A	E_{mag}/E_{mag}^{Ohm}	E_{kin}/E_{mag}	Π_{mag}	M_{mag}	symmetry
0.1	1.2	0.96	0.018	1	0	**S0**
	0.12	1	0.013	1	0	**S0**
1	12	0.013	0.26	0.524	0.8876	mixed
	1.2	0.932	0.0011	1	0	S0
	0.12	0.999	0.00005	1	0	**S0**
10	0.12	0.998	0.00035	1	0	**S0**

- toroidal field easier to stabilize than poloidal
- increasing $Pm \longrightarrow$ harder to stabilize

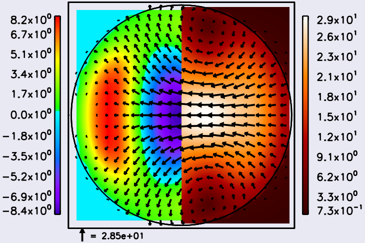
Toroidal Background Field: Final Fields

Stable: $P = 6$ ms





Unstable: $P = 600$ ms



Discussion

- stable purely toroidal equilibrium found
  $\Rightarrow$ twisted torus not needed
- slow rotation
  $\Rightarrow$ drastic field reduction
  "final" field: poloidal+toroidal

Conclusions

- neutron stars born with $B \gtrsim 10^{15}$G:
  - those with $P \lesssim 5$ ms AND $\alpha \lesssim 45^o \Rrightarrow$ magnetars
  - those with $P \gtrsim 10$ ms AND/OR $\alpha \gtrsim 45^o \Rrightarrow$ standard pulsars
  - No other stabilizing effect (torus–like ***B*** configuration, density stratification), only **RAPID ROTATION** can maintain magnetar field!
- toroidal fields easier to stabilize than poloidal ones
- hints of existence of unique stable dipolar/toroidal equilibria
- neutron stars born with $B \sim 10^{12} \ldots 10^{13}$G, $P \leq 60$ ms: always stabilized!