Neutrino-Nucleon Scattering Rates in Proto-Neutron Stars

L. Mornas

Departamento de Física, Universidad de Oviedo, Spain

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$\boxed{1}$ MOTIVATIONS

▷ From supernovae to neutron stars:

- neutrinos carry $\sim$ 99% of energy released
- "delayed" mechanism = shock revival? (Wilson 1985)
- neutrino transport = necessary (but not sufficient) ingredient

▷ Neutrino burst:

- peak:
  recombination (collapse) $\sim$ 10 ms
- shoulder:
  trapping (accretion phase) $\sim$ 500 ms
  solve Boltzmann equation
  $\leftrightarrow$ differential cross section needed
- tail:
  deleptonization (PNS cooling) $\sim$ 20-50 s
  solve diffusion equation
  $\leftrightarrow$ mean free path needed
  possible phase transitions
  (quarks, condensates..)
  metastability & collapse to BH
- after 50s: star transparent to neutrinos
  cooling curve $\sim 10^6$ yrs
  total neutrino emissivity needed

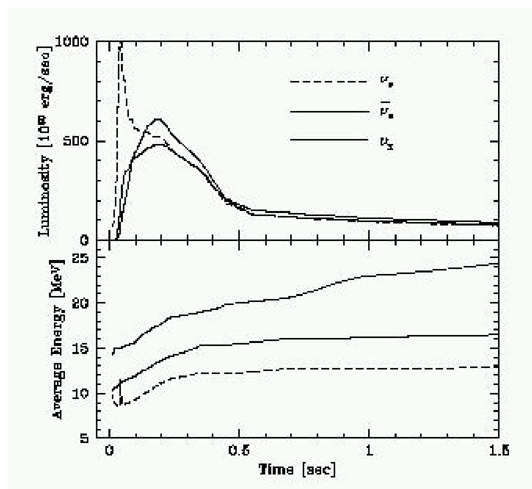

Fig. 1 – Neutrino burst
[T. Totani, H.E. Dalhed, K. Sato, J.R. Wilson, Ap.J. **496** (1998) 216]

▷ Some references for this section:

[1] H.T. Janka, Astron. Astrophys. **368** (2001) 560

[2] A. Burrows, S. Reddy, T.A. Thompson, astro-ph/0404432

[3] J. Pons, S. Reddy, M. Prakash, J.M. Lattimer, J.A. Miralles, Astrophys. J. **513** (1999) 780

Various processes

Several reactions contribute, depending on the density and temperature

▷ In the core = proto neutron star:

($\rho > \rho_0 = 2\ 10^{14}$ g.cm^3) diffusion, production and absorption in reactions with unbound nucleons (also hyperons or quarks after PNS has cooled somewhat)

$$\text{charged current:} \quad \begin{cases} p + e^- \to \nu_e + n \\ n \to p + e^- + \overline{\nu}_e \end{cases}$$

$$\text{neutral current:} \quad \nu_\ell + N \to \nu_\ell + N \qquad \text{with } \nu_\ell \in \{\nu_e, \overline{\nu}_e, \nu_\mu, \overline{\nu}_\mu, \nu_\tau, \overline{\nu}_\tau\}\ , N \in \{n, p\}$$

▷ Between the PNS and the neutrinosphere:

$$\begin{array}{rl} \text{Diffusion} & \nu_\ell + N \to \nu_\ell + N \\ \text{Mod. URCA} & \nu_\ell + N + N \leftrightarrow \ell_\ell + N + N' \\ \text{Bremsstrahlung} & N + N \to N + N + \nu_\ell + \overline{\nu}_\ell \\ \text{Pair annihilation} & e^- + e^+ \to \nu_\ell + \overline{\nu}_\ell\ , \qquad \nu_\ell + \overline{\nu}_\ell \to \nu_{\ell'} + \overline{\nu}_{\ell'} \\ \ldots & \end{array}$$

Evolution of proto neutron star

Characterized by finite temperature and neutrino content

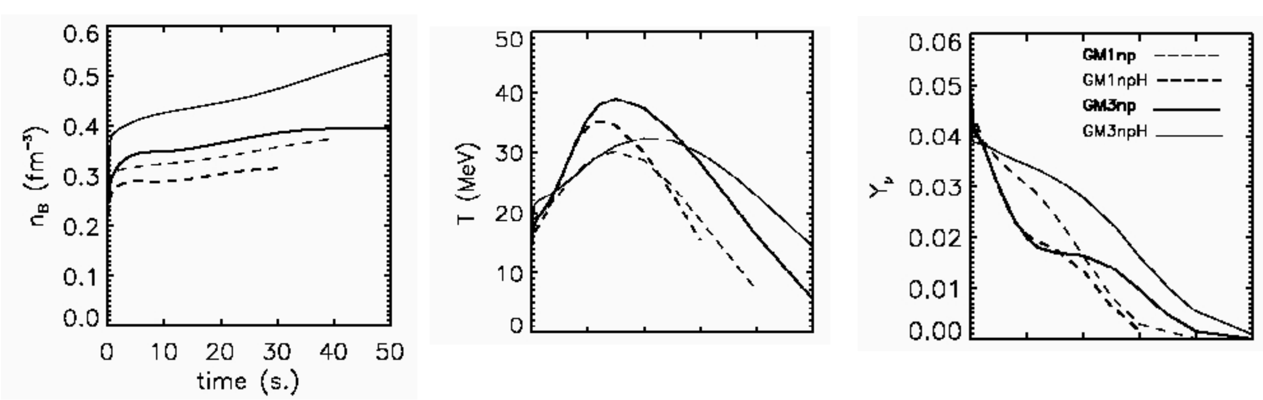

Fig. 2 – Evolution of central density, temperature and neutrino content of a protoneutron star [from J.A. Pons, *et al.*, Ap.J. **513** (1999) 780]

Asymmetric nuclear matter

The proton fraction $Y_p = \dfrac{\rho_p}{\rho_p + \rho_n}$ is determined by the β equilibrium condition $\hat{\mu} = \mu_n - \mu_p = \mu_e - \mu_\nu$. If neutrinos are trapped inside matter, the chemical potential of the neutrino has a finite value $\mu_\nu = (6\pi^2 \rho Y_\nu)^{1/3}$. The lepton fraction $Y_L = Y_\nu + Y_e$ is determined by neutrino transport (eg, diffusion equation) to which the neutrino-nucleon cross section serves as input. A typical value is $Y_L \simeq 0.4$

- In cool neutron stars,
  neutrinos leave the star unhindered $\longrightarrow Y_p \simeq 0.1$
- In supernovae and protoneutronstars,
  neutrinos are trapped by high density and temperature $\longrightarrow Y_p \simeq 0.3$

Cross section

$$\text{Consider} \qquad \nu(K) + n(P) \to \nu(K') + n(P')$$

$$H = \frac{G_F}{\sqrt{2}} J_\alpha \ell^\alpha \quad \to \quad d\sigma \sim \frac{G_F^2}{2} \sum J_\alpha^* J_\beta \ell^{*\alpha} \ell^\beta \quad \to \quad d\sigma \sim \frac{G_F^2}{2} \, Im\,(S_{\mu\nu} L^{\mu\nu})$$

- *Im*: put particles on their mass shell
- $L_{\mu\nu} = Tr\,[\gamma.K \gamma_\mu (1-\gamma_5) \gamma.K \gamma_\mu (1-\gamma_5)]$: lepton current
- $S^{\mu\nu} = \int dx < J^\mu(x) J^\nu(0) > \propto \Pi^{\mu\nu}$: hadron current in dense medium
  $\to$ Structure function (Sum on all momenta of nucleons, include blocking factors, take into account correlations)

▷ Orders of magnitude:

$$\text{Cross section} \qquad \sigma(\nu N \to \nu N) \sim (c_v^2 + 3c_A^2) \frac{G_F^2}{\pi} E_\nu^2 \simeq 2\ 10^{-40}\ \text{cm}^2 \left(\frac{E_\nu}{100\ \text{MeV}} \right)^2$$

$$\text{density of nucleons} \quad n_B \simeq \frac{\rho_N}{m_N} \simeq 2\ 10^{38}\ \text{cm}^{-3}$$

$$\text{mean free path} \qquad \lambda \simeq (\sigma n_B)^{-1} \simeq 28\ \text{cm} \left(\frac{100\ \text{MeV}}{E_\nu} \right)^2$$

$$\text{diffusion time} \qquad \frac{R^2}{\lambda} \simeq 1.2\ s \left(\frac{R}{10\ \text{km}} \right)^2 \left(\frac{E_\nu}{100\ \text{MeV}} \right)^2$$

In-medium corrections

In dense matter, neutrino scattering with nucleons is modified by correlations with the medium. Several effects contribute:

- Blocking factors $(1-f) \leftrightarrow$ is final state available for scattering
- Effective nucleon masses, mean potential created by other nucleons (mean field aprox.)
- Hartree-Fock correlations: σ reduced by ~ 20 % reduction according to [Fabbri&Matera '96, Niembro *et al.* '01]
- RPA correlations: Studied on diffusion, production / absorption processes.
  see *e.g.* [relativistic] $\rightarrow$ Reddy *et al* '99, Yamada & Toki '00, Mornas& Pérez '02
  [non-relat.] $\rightarrow$ Navarro *et al.* '99, Margueron '01, '03, Shen *et al.* '03, Mornas '05, '06
  $\Rightarrow$ Reduction of cross section except if a collective mode is excited
- Short range correlations *e.g.* Brueckner-Hartree-Fock
  partially taken into account by use of phenomenological interactions
  Full effect studied on Bremsstrahlung & modified URCA
  $\Rightarrow$ Nucleons acquire a "width" $\Gamma \rightarrow$ multiple scattering corrects infrared divergence $1/\omega^2 \rightarrow 1/(\omega^2+\Gamma^2/4)$ (Landau-Migdal-Pomeranchuk effect)
  see e.g. Raffelt&Seckel '91, Hannestadt&Raffelt '98, Sedrakian&Dieperink '00, Van Dalen '03
- Axial quenching $g_A = 1.26 \rightarrow g_A \sim 1$
- Vertex corrections "weak magnetism" [Horowitz '97] $\Gamma^\mu_W = \gamma^\mu(g_V - g_A\gamma_5)$
  $\rightarrow \widetilde{\Gamma}^\mu_W = F_1(Q^2)\gamma^\mu + iF_2(Q^2)\sigma^{\mu\nu}q_\nu/(2M) + G_A(Q^2)\gamma_5\gamma^\mu + iF_p(Q^2)\gamma_5 q^\mu$
- Phase diagram of dense matter, exoticas
  (hyperons, quarks, meson condensates, color superfluidity, ...)
- Coherent scattering on ordered phases
  (in crust, or exotic phases in the center)
  see e.g. Reddy *et al.* '00, Horowitz *et al.* '04, '05
- Strong magnetic fields

Random phase approximation and onset of instabilities

It is generally found that the neutrino opacities are suppressed by medium effects. If, however, a collective mode is excited, there is a sizeable enhancement. Here we consider RPA calculations in order to investigate the possible existence of such modes.

▷ Dyson equation:

The polarization appearing in the expression of the cross section (in the structure function) is taken as the sum of an infinite series of one-loop insertions. Schematically one has

$$\Pi^{RPA} = \Pi_0 + \Pi^{RPA} V \Pi_0 \qquad \text{(Dyson equation)}$$

V is the nuclear interaction. Diagrammatically,

▷ Repulsive *vs.* attractive interactions:

The Dyson equation can be formally solved

$$\Pi^{RPA} = \frac{\Pi_0}{1 - V\Pi_0}$$

$$Im\Pi^{RPA} = \frac{Im\Pi_0}{(1 - V Re\Pi_0)^2 + (V Im\Pi_0)^2} = \frac{Im\,\Pi_0}{\epsilon}$$

In the denominator appears the dielectric function ϵ ($\leftrightarrow$ dispersion relation).

- Repulsive interaction: $\epsilon > 1 \rightarrow$ reduction of cross section $\sigma \propto Im\,\Pi^{RPA}$
- Attractive interaction: $\epsilon < 1 \rightarrow$ enhancement of cross section.

If there is a collective mode ϵ vanishes $\rightarrow$ there is a pole in the cross section

▷ Instabilities and enhancement of cross section:

→ A mechanical instability appears in response to density fluctuations in the vector channel at subnuclear density, corresponding to clustering in the crust (nuclei + fluid of dripped neutrons)
[*cf.* analogy: critical opalescence in a liquid / solid transition]

→ A spin instability may appear in nonrelativistic (Skyrme) as well as relativistic ($\sigma\omega\rho$) models of the nuclear interaction
[see e.g.: non-rel. → Kutschera (1994), Margueron (2001), Isayev (2004) , *etc*
relat. → Bernardos (1995), Maruyama (2001)]

It has been suggested that this is related to Hartree-Fock contribution

$\boxed{3}$ RELATIVISTIC CALCULATIONS

▷ Some references for this section:

[1] C.J. Horowitz and K. Wehrberger, Nucl. Phys. **A531** (1991) 66
H. Kim, J. Piekarewicz, C.J. Horowitz, Phys. Rev. C51 (1995) 2739

[2] S. Reddy, M. Prakash, J.M. Lattimer and J. Pons, Phys. Rev. **C59** (1999) 2888

[3] S. Yamada and H. Toki, Phys. Rev. **C61** (2000) 015803

[4] L. Mornas, A. Pérez, Eur. Phys. J **A13** (2002) 383;
L. Mornas , Nucl. Phys. **A721** (2003) 1040

Interaction Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm int} \;=\; & \overline{\psi}\left(-g_\sigma \sigma + g_\omega \gamma^\mu \omega_\mu - \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu \partial_\mu \vec{\pi}.\vec{\tau} - g_\delta \vec{\delta}.\vec{\tau} + g_\rho \gamma^\mu \vec{\rho}_\mu.\vec{\tau} + \frac{f_\rho}{2M}\sigma^{\mu\nu}\partial_\nu \vec{\rho}_\mu.\vec{\tau}\right)\psi \\ & -\frac{1}{3} b m_N \sigma^3 - \frac{1}{4} c\sigma^4 + \frac{1}{4} d\left(\omega_\alpha \omega^\alpha\right)\end{aligned}$$

• Non-linear σ and ω couplings σ^3, σ^4, ω^4 are introduced in order to obtain a better description of the incompressibility modulus and effective mass at saturation.

• Tensor coupling of the ρ meson plays an important role for NN scattering and at RPA level.

• The δ meson can be important in p-n asymmetric matter (gives $M_n^* \neq M_p^*$)

• Residual contact interaction introduced to correct the short range behavior of the pion potential (avoids spurious zero sound branch in the pion dispersion relation). We add a contact term; this leads to the usual replacement

$$\mathcal{L} \ni g'\left(\frac{f_\pi}{m_\pi}\right)^2 \left(\overline{\psi}\gamma_5\gamma_\mu\psi\right)\left(\overline{\psi}\gamma_5\gamma^\mu\psi\right) \qquad \Rightarrow \qquad \Pi_\pi \;\longrightarrow\; \frac{q^2\Pi_\pi}{q^2 - g'\Pi_\pi}$$

Differential cross section

$$\frac{d\sigma}{dE_\nu d\Omega} = \frac{G_F^2}{64\pi^3}\frac{E_{\nu'}}{E_\nu}\mathcal{I}m(S^{\mu\nu}L_{\mu\nu})$$

• E_ν is the ingoing and $E_{\nu'} = E_\nu - \omega$ is the outgoing lepton energy. The energy loss ω is the zero component of momentum exchange $q^\mu = K^\mu - K'^\mu$.
• $L_{\mu\nu}$ is the lepton current
• The structure function $S^{\mu\nu}$ can be related to the imaginary part of the retarded polarization

$$S^{\mu\nu}(q) \;=\; \int d^4x\; e^{iq.x} \; < J^\mu(x)J^\nu(0) >= \frac{-2}{1-e^{-z}}\; \mathcal{I}m\Pi_R^{\mu\nu}$$

• The factor $(1-e^{-z})^{-1}$ with $z = \beta(\omega - \Delta\mu)$ arises from detailed balance. $\Delta\mu$ is the difference between the chemical potential of the outgoing and ingoing nucleons.

▷ For free scattering

$$\mathcal{R}e\;\Pi_R^{\alpha\beta} = \mathcal{R}e\;\Pi_{11}^{\alpha\beta} \quad , \qquad \mathcal{I}m\;\Pi_R^{\alpha\beta} = \tanh\left(\frac{\beta\omega}{2}\right)\mathcal{I}m\;\Pi_{11}^{\alpha\beta}$$

$$\Pi_{11}^{\alpha\beta} = -i\int d^4p\;\mathrm{Tr}\left[\Gamma^\alpha G^{11}(p)\Gamma^\beta G^{11}(p+q)\right]$$

with $\Gamma^\alpha = \gamma^\alpha(C_V - C_A\gamma_5)$ being the weak vertex to the hadronic current.
$G^{11}(p)$ is the nucleon propagator.

▷ At the mean field level

$$G^{11}(p) \;=\; (\gamma.P + M)\left\{\frac{1}{P^2 - M^2 + i\epsilon} + 2i\pi\delta(P^2 - M^2)\left[\theta(p_0)n(p_0) + \theta(-p_0)\overline{n}(p_0)\right]\right\}$$

$$\text{with} \qquad n(p_0) = \frac{1}{e^{\beta(p_0-\mu)}+1}\;, \quad \overline{n}(p_0) = \frac{1}{e^{-\beta(p_0-\mu)}+1}$$

where M is the effective nucleon mass.

▷ RPA correlations

RPA correlations are introduced by substituting the mean field polarization by the solution of the Dyson equation

$$\begin{aligned}\widetilde{\Pi}^{\mu\nu}_{WW} &= \Pi^{\mu\nu}_{WW} + \sum_{a,b=\sigma,\omega,\rho,\pi\ldots} \Pi^{(a)\,\mu\alpha}_{WS} D^{(ab)}_{SS\,\alpha\beta} \widetilde{\Pi}^{(b)\,\beta\nu}_{SW} \\ &= \Pi^{\mu\nu}_{WW} + \sum_{a,b=\sigma,\omega,\rho,\pi\ldots} \Pi^{(a)\,\mu\alpha}_{WS} \widetilde{D}^{(ab)}_{SS\,\alpha\beta} \Pi^{(b)\,\beta\nu}_{SW}\end{aligned}$$

W or S: vertex with a weak or strong coupling respectively. D_{SS} is the propagator of the mesons $a = \sigma,\ \omega,\ \rho^0\ldots$ which mediate the nuclear interaction. $\widetilde{D}_{SS}$ is the meson propagator dressed in the RPA approximation. The first term of the Dyson equation corresponds to the mean field approximation.

Explicitely (with full meson mixing in propagator matrix):

$$\Delta\Pi^{RPA} = \begin{pmatrix} \Pi^{(\sigma)\mu}_{WS} & \Pi^{(\omega)\mu\alpha}_{WS} & \Pi^{(\delta)\mu}_{WS} & \Pi^{(\rho)\mu\alpha}_{WS} \end{pmatrix} \times \begin{pmatrix} G^{\sigma} & G^{\sigma\omega}_{\mu} & G^{\sigma\delta} & G^{\sigma\rho}_{\mu} \\ G^{\omega\sigma}_{\nu} & G^{\omega\omega}_{\mu\nu} & G^{\omega\delta}_{\nu} & G^{\omega\rho}_{\mu\nu} \\ G^{\delta\sigma} & G^{\delta\omega}_{\nu} & G^{\delta\delta} & G^{\delta\rho}_{\nu} \\ G^{\rho\sigma}_{\nu} & G^{\rho\omega}_{\mu\nu} & G^{\rho\delta}_{\nu} & G^{\rho\rho}_{\mu\nu} \end{pmatrix} \times \begin{pmatrix} \Pi^{(\sigma)\nu}_{SW} \\ \Pi^{(\omega)\beta\nu}_{SW} \\ \Pi^{(\delta)\nu}_{SW} \\ \Pi^{(\rho)\beta\nu}_{SW} \end{pmatrix}$$

Decomposition of polarization

The polarizations $\widetilde{\Pi}^{\mu\nu}_{WW} = \Pi^{\mu\nu}_{WW} + \Delta\Pi^{\mu\nu}_{\mathrm{RPA}}$ which enter the definition of the differential neutrino-nucleon scattering cross section may be decomposed onto orthogonal projectors

$$\widetilde{\Pi}^{\mu\nu}_{WW} = \widetilde{\Pi}_T\ T^{\mu\nu} + \widetilde{\Pi}_L\ \Lambda^{\mu\nu} + \widetilde{\Pi}_Q\ Q^{\mu\nu} + i\,\widetilde{\Pi}_E\ E^{\mu\nu}$$

$$\begin{aligned} \text{with}\qquad \Lambda^{\mu\nu} &= \frac{\eta^\mu\eta^\nu}{\eta^2} \quad ; \quad \eta^\mu = u^\mu - \frac{q.u}{q^2}q^\mu \quad && \text{longitudinal} \\ T^{\mu\nu} &= g^{\mu\nu} - \frac{\eta^\mu\eta^\nu}{\eta^2} - \frac{q^\mu q^\nu}{q^2} && \text{transverse} \\ E^{\mu\nu} &= \epsilon^{\mu\nu\rho\lambda}\eta_\rho q_\lambda && \text{axial} \\ Q^{\mu\nu} &= \frac{q^\mu q^\nu}{q^2} && \text{does not contribute} \end{aligned}$$

where $g^{\mu\nu} = \mathrm{diag}\,(1,-1,-1,-1)$, u^μ=hydrodynamic velocity, q^μ=transferred momentum.

Response functions

The contraction of the lepton current with the polarization

$$-2\frac{\mathcal{I}m\left(L^{\mu\nu}\Pi^R_{\mu\nu}\right)}{1-e^{-z}} = 4E_\nu E_{\nu'}\left[R_1(1+\cos\theta)+R_2(3-\cos\theta)-2(E_\nu+E_{\nu'})R_5(1-\cos\theta)\right]$$

can be expressed by means of three structure functions R_1, R_2 and R_5 related to the previous polarizations by

$$\begin{aligned} R_1 &= \frac{-2}{1-e^{-z}}\mathcal{I}m\left[-\frac{q^2}{\mathbf{q^2}}\Pi_L+\frac{w^2}{\mathbf{q^2}}\Pi_T\right] \\ R_2 &= \frac{2}{1-e^{-z}}\mathcal{I}m\left[\Pi_T\right] \\ R_5 &= \frac{2}{1-e^{-z}}\mathcal{I}m\left[\Pi_E\right] \end{aligned}$$

In the non relativistic limit, R_1 and R_2 reduce to the density and spin density correlation functions respectively. The axial-vector structure function R_5 appears only in a relativistic treatment.

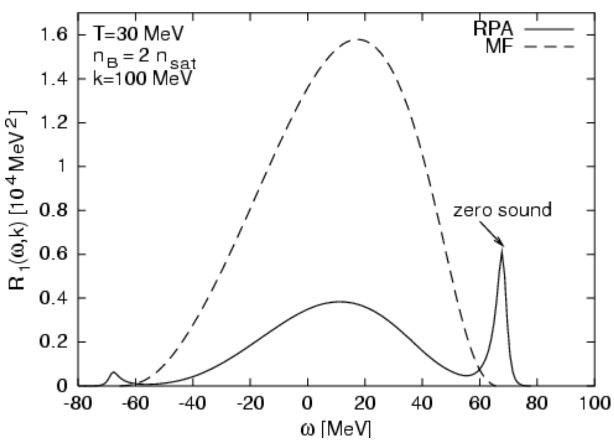

Fig. 3-a – Structure function R1, displaying overall RPA reduction and zero sound enhancement

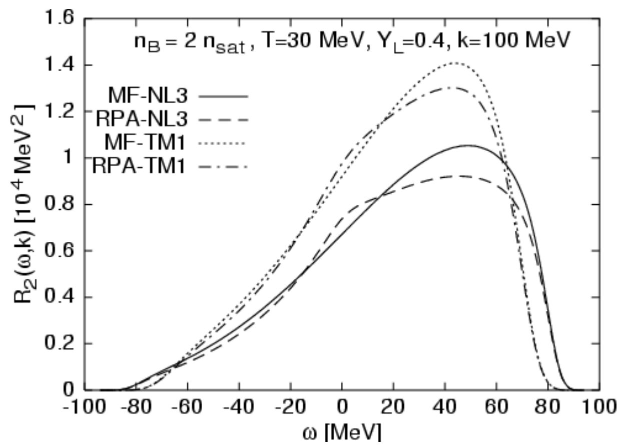

Fig. 3-b – Structure function R2- Mean field and RPA results for two standard parameter sets

Mean free path

An *estimate* of the mean free path is obtained by integrating the differential cross section (other processes contribute!)

$$\frac{1}{\lambda(E_\nu)} = -\frac{G_F^2}{32\pi^2}\frac{1}{E_\nu^2}\int_0^\infty q dq \int_{-k}^{\omega_{\max}} d\omega \frac{(1-f(E'_\nu))}{1-e^{-z}} \mathcal{I}m \left(L^{\alpha\beta}\Pi^R_{\alpha\beta}\right)$$
$$\text{with} \quad \omega_{\max} = \min(2E_\nu - k, k)$$

$(1-f(E'_\nu)) = (1+\exp[(E'_\nu-\mu_\nu)/T]))$ is a Pauli blocking factor for the outgoing lepton. The chemical potential μ_ν is determined by β equilibrium and neutrino transport (eg, diffusion equation).

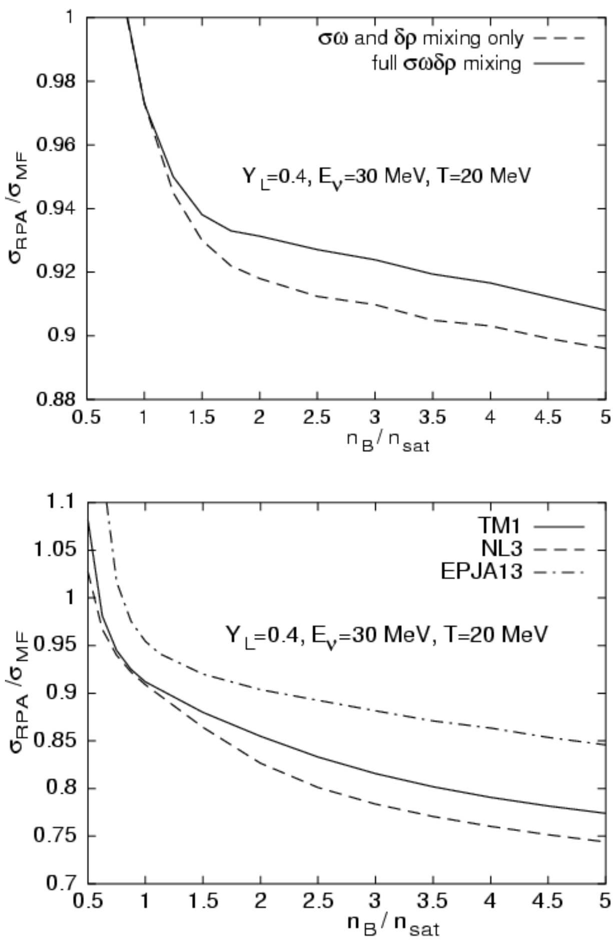

Fig. 4 – Reduction factor in matter with trapped neutrinos:
(upper panel) Effect of meson mixing
(lower panel) Comparison of various parametrizations

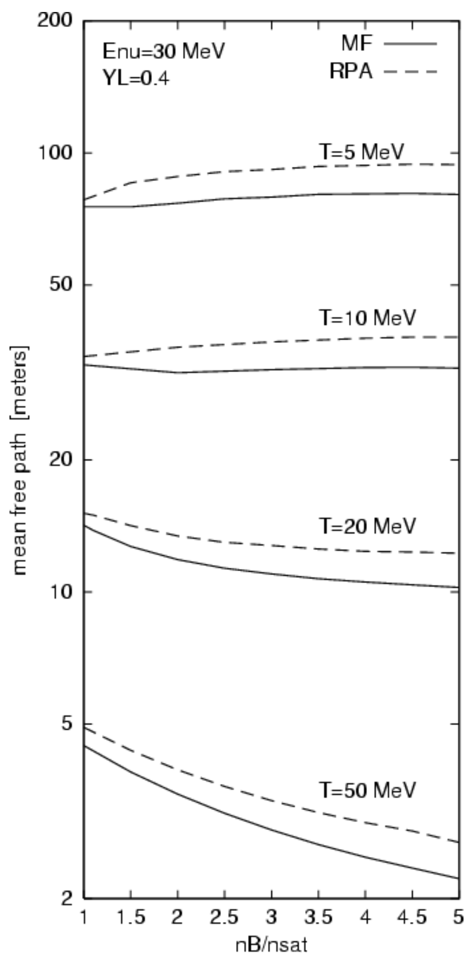

Fig. 5 – Mean free path in matter with trapped neutrinos

Main results

• The "vector" structure function R_1 involves the sigma meson and the longitudinal part of the omega and rho mesons. A zero sound mode in the mixed σ-ω dispersion relation manifests itself as a pole in the RPA meson propagator. However it lies in a marginally accessible range (at high energy tranfer). It is moreower quenched by Landau damping. It doe not contribute appreciably to the mean free path.

• The transverse contribution is dominant and provides for about 60 % of the total result. It is very little modified by RPA correlations. The corrections arise from the subdominant longitudinal and axial-vector polarizations Π_L and Π_E.

• In fact the pion does not contribute directly to the neutral current process. The dependence of the structure functions for the diffusion (neutral process) on the strength of residual interactions in the tensor channel (Landau-Migdal parameter g') is very small (when treated covariantly).

• At high density, the total neutrino-neutron scattering cross section is found to be reduced by RPA correlations by a factor 10% to 25% with respect to the mean field result.

• Parameter sets adjusted to give a lower effective mass at saturation yield a stronger reduction factor.

• At low density and moderate temperature, on the other hand, RPA correlations would yield an enhancement; however the validity of the model becomes questionable in this range.

• When full meson mixing is taken into account, the reduction is smaller.

• The coupling to the δ meson enhances the longitudinal response with respect to the case g_δ=0, whereas it suppresses the transverse response.

$\boxed{4}$ NON RELATIVISTIC CALCULATIONS

▷ Some references for this section:

[1] N. Iwamoto and C.J. Pethick, Phys. Rev. **D25** (1982) 313

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[7] L. Mornas, Eur. Phys. J. **A27** (2006) 49

Neutrino-nucleon scattering rate

Differential cross section in non relativistic approximation:

$$\frac{1}{V}\frac{d\sigma}{d\omega d\Omega} = \frac{G_F^2}{8\pi^3}(E'_\nu)^2[1-f(E'_\nu)]\left[(1+\cos\theta)\mathcal{S}^{(0)} + (3-\cos\theta)\mathcal{S}^{(1)}\right]$$

In this limit, the spin 0 and spin 1 responses correspond to the vector and axial couplings respectively.

$$\begin{aligned}
\mathcal{S}^{(0)} &= \mathcal{S}_V(k,\omega) \sim \int e^{-i\omega t+ik.x} < n(x,t)n(0) > \qquad \text{density correl/fluct} \\
\mathcal{S}^{(1)} &= \mathcal{S}_A(k,\omega) \sim \int e^{-i\omega t+ik.x} < j^i(x,t)j^j(0) > \qquad \text{spin-density correl/fluct}
\end{aligned}$$

If no instabilities are excited, the main contribution comes from the axial response function

In the RPA, vector and axial structure functions obey Dyson equations:

$$\Pi^{RPA}_{V/A} = \Pi_0 + \Pi^{RPA}_{V/A}\, V_{(S=0/1)}\, \Pi_0,$$

In asymmetric np matter in β equilibrium, 2x2 matrix structure,

$$e.g.\ \Pi_0 = \begin{bmatrix} \Pi^0_n & 0 \\ 0 & \Pi^0_p \end{bmatrix}.\ \text{(Linhardt function)}$$

Interaction potential in the p-h channel: Landau Fermi Liquid approximation

$$V_{(S=0)} = \begin{bmatrix} f_{nn} & f_{np} \\ f_{pn} & f_{pp} \end{bmatrix} \qquad V_{(S=1)} = \begin{bmatrix} g_{nn} & g_{np} \\ g_{pn} & g_{pp} \end{bmatrix}$$

Landau parameters

- interaction between particles having momentum $\simeq$ Fermi momentum
- obtained from second functional derivative of polarized energy functional w.r.t the density (or from first derivative of single particle potential)

Monopolar approximation (l=0): no angular or momentum dependence, Landau parameters=real functions of the partial densities of neutrons and protons.

Spin (in-)stability

Criterion: a spin instability occurs when the determinant of the inverse magnetic susceptibility matrix (χ_{ij} where $i, j \in \{n, p\}$) vanishes. In terms of the Landau parameters:

$$\mathrm{Det}\left(\frac{1}{\chi_{ij}}\right) = 0 \leftrightarrow \mathrm{Det}\begin{pmatrix} (1+G_0^{nn}) & G_0^{np} \\ G_0^{pn} & (1+G_0^{pp}) \end{pmatrix} = 0$$

where $G_0^{ij} = \sqrt{N_0^i N_0^j} g_0^{ij}$, $N_0^i = m_i^* k_{Fi}/\pi^2$

On the other hand, solving the RPA equations, we obtain *e.g.*

$$\Pi^{\mathrm{RPA}}_{A\,nn} = \frac{(1 - g_{pp}\Pi_p^0)\Pi_n^0}{\mathrm{Det}[I - V_{(S=1)}\,\Pi_0]} \propto \frac{1}{D_A}$$

In the limit where the temperature T and the energy transfer ω go to zero we find

$$\mathcal{R}\mathrm{e}\,\Pi_0^i(\omega, k) \xrightarrow{(T\to 0,\,\omega\to 0,\,\omega/k=\mathrm{cst})} -N_0^i \Rightarrow D_A = \mathrm{Det}[I - V_{(S=1)}\,\Pi_0] = 0 : \mathrm{pole}$$

$\rightarrow$ peak in the structure function and neutrino cross section

Effective nuclear interactions

▷ Skyrme interaction

$$V_{NN}(r) = t_0\,(1+x_0P_\sigma)\delta(r)+\frac{1}{2}t_1\,(1+x_1P_\sigma)\left[k'^2\ \delta(r)+\delta(r_1-r_2)\ k^2\right]$$
$$+t_2\,(1+x_2P_\sigma)k'\,\delta(r)\,k+\frac{1}{6}t_3\,(1+x_3P_\sigma)\,\rho_N^\alpha\delta(r),$$

→ good description of properties of nuclei, nuclear matter and neutron stars.
→ recent parametrizations (SLy: Chabannat *et al.*, 1998) adjusted to reproduce microscopical neutron matter calculations

▷ Gogny interaction

$$V(r_{12}) = \sum_{i=1,2}\left(t_i^W+t_i^BP_\sigma-t_i^HP_\tau-t_i^MP_\sigma P_\tau\right)\ \exp-(r_{12}/a_i)^2$$
$$+\sum_{i=1,2}t_{3i}(1+x_{3i}P_\sigma)\rho^{\alpha_i}(R)\delta(r_{12})$$

→ finite range + contact term to describe correlations
→ good description of properties of nuclei and nuclear matter
→ original set D1S not suitable for neutron star matter, improved parametrization D1P (Farine et al, 1999) adjusted to reproduce microscopical neutron matter calculations

▷ Modified Seyler-Blanchard interaction (MSB)

Originally introduced by Myers and Swiatecki (1969)
→ finite range (Yukawa), momentum and density dependent
→ good description of properties of nuclei, nuclear matter and neutron stars [Myers & Swiatecki (1990,1996), Bandyopadhyay (1990), Strobel (1997)]
→ used to describe polarized nuclear matter in the context of neutron stars [Uma Maheswari (1997, 1998)]

$$V=-C_{ul}\left[1-\frac{p^2}{b^2}-d^2(\rho_1+\rho_2)^n\right]\frac{e^{-r/a}}{r/a}$$

C_{ul} for $\{n\uparrow,n\downarrow,p\uparrow,p\downarrow\}$

▷ Density-dependent Michigan 3-range interaction interaction (M3Y)

Central part of the interaction can be decomposed as

$$V(r_{12}) \;=\; \sum_i \left(t_i^W + t_i^B P_\sigma - t_i^H P_\tau - t_i^M P_\sigma P_\tau\right)\; f_i(r_{12})$$

The Gogny forces use as the $f_i(r)$ functions two gaussians whereas the M3Y use three Yukawas $f_i(r) = \exp(-\mu_i\, r)/(\mu_i\, r)$.

$\rightarrow$ Three ranges corresponding to Compton lengths of σ, ω and π mesons
$\rightarrow$ Based upon G-matrix elements of the Paris potential
$\rightarrow$ Density dependence introduced in 2 different ways:
multiply by a density-dependent scale factor $\mathcal{F}(\rho)$ [Khoa (1996,)] ...
... or add a density dependent contact interaction (Skyrme and Gogny-like) [Nakada (2003)]
$\rightarrow$ good description of properties of nuclei
$\rightarrow$ used to study neutron rich nuclei
$\rightarrow$ used to describe low energy nucleus-nucleus collisions

▷ Parametrization of microscopical calculations

$\rightarrow$ VB - ZLS

Brückner-Hartree-Fock calculation of polarized nuclear matter by Vidaña & Bombaci (2002) =“VB”, and of Landau parameters with application to neutrino-nucleon cross section by Margueron *et al.* (2003) and Zuo *et al.* and Shen *et al.* (2002, 2003) = “ZLS”

$\rightarrow$ APR+CP

Parametrization of variational calculations by Akmal, Pandharipande and Ravenhall (APR98) for spin saturated systems, and extension by Cowell and Pandharipande (2004) =“CP” for spin polarized systems. (NB: transition to pion condensed phase)

Behavior of the Landau parameters and stability criterion in the spin channel

▷ In spin-isospin channel ("Gamow Teller"):

→ The microscopical calculations (Brueckner-Hartree-Fock from ZLS03 and variational from APR+CP)

• reproduce the experimental value at saturation density determined from observation of the Gamow Teller giant resonance. $G'_0 \sim 1.18$ [Suzuki, 1999]

• coincide for $n_B < n_{\mathrm{sat}}$ but differ strongly at high density

→ The M3Y interactions are also compatible with the experimental value at saturation density

→ Skyrme interaction does not generally reproduce the experimental G'_0

▷ In spin channel ("Fermi"):

→ The parametrization of variational results APR+CP also reproduces the recommended value $G_0 = 0.1 \pm 0.1$

→ BHF results in contrast obtain too high values

→ Skyrme results in general too high at saturation density

→ The MSB result is compatible with data

→ M3Y models are again compatible with the experimental constraint

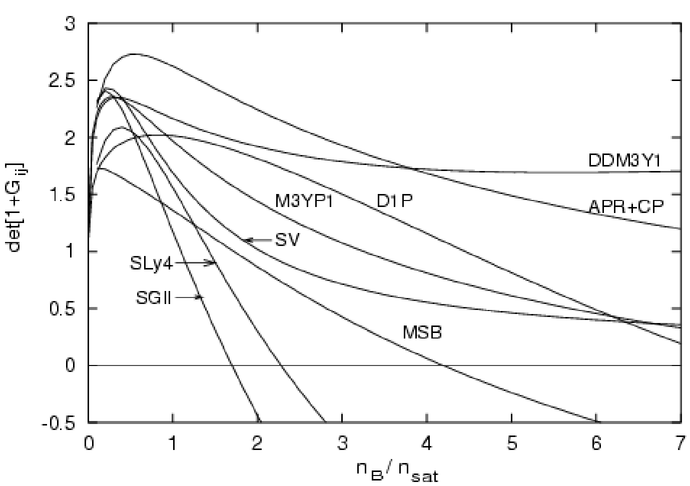


Fig. 6 – Stability criterion in the spin channel, matter in β-equilibrium

Response functions

▷ Spin zero sound:

microscopical (variational or Brueckner) or M3Y, Gogny at low density

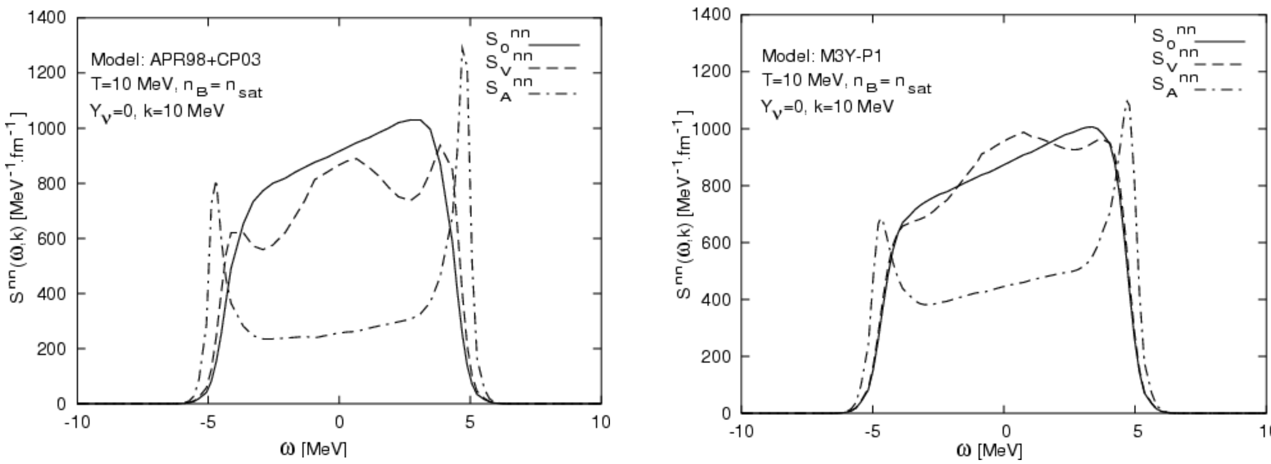


Fig. 7 – Neutron contribution to the axial response function for matter in β-equilibrium at saturation density, from the APR98+CP03 and M3Y-P1 parametrizations

▷ Spin instability :

Skyrme or MSB, Gogny at high density

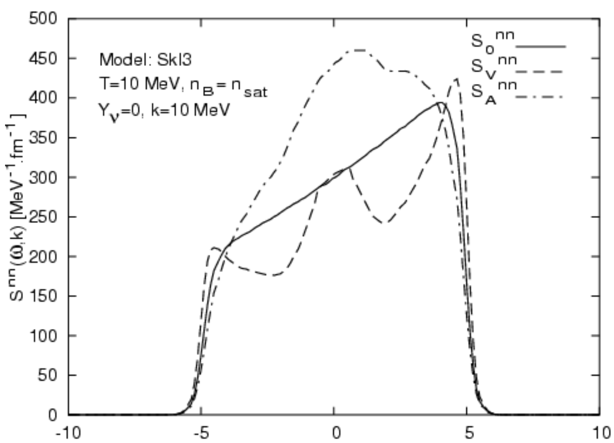


Fig. 8 – Exemple of development of spin instability. Here for Skyrme model SkI3 at saturation density.

Mean free path for various non relativistic models

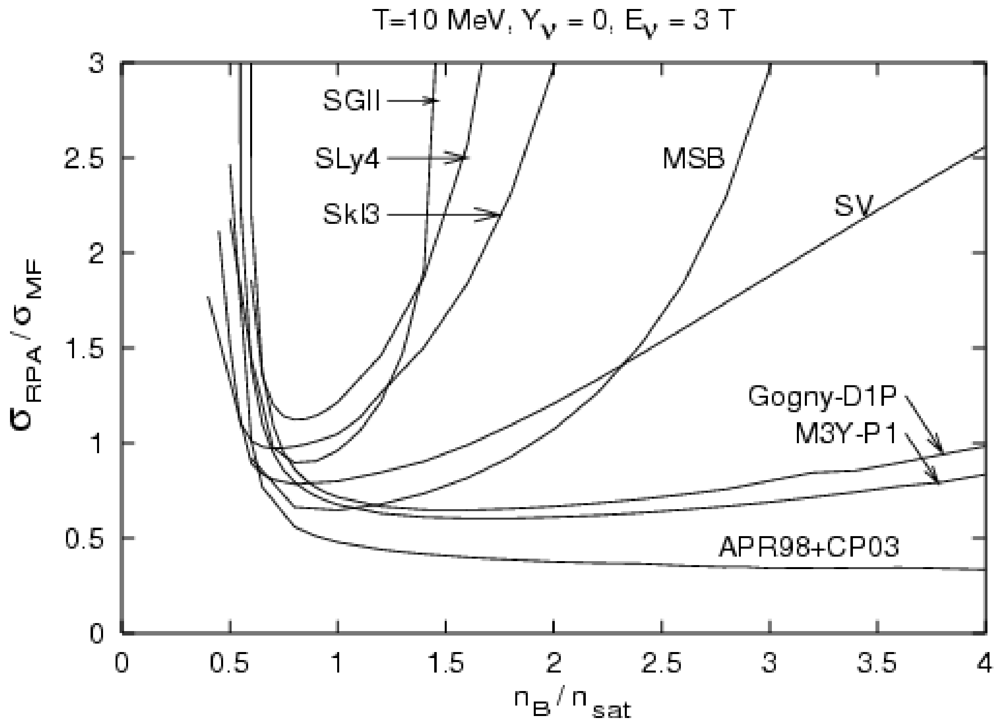


Fig. 9 – RPA correction to the cross section for various models

→ Skyrme models (all except SV) or modified Seyler Blanchard (MSB) display enhancement of the cross section in the RPA w.r.t. Hartree-Fock calculation, and divergence in the vicinity of spin instability. Transition occurs later for MSB.

→ Skyrme SV, stable for spin excitations, still displays an enhancement (by a factor of 2 at $3\rho_{sat}$)

→ Microscopical models (BHF or variational) predict a reduction of the cross section from RPA correlations: at $3\rho_{sat}$, by a factor of 2 for ZLS03, factor of 3 for APR98+CP03, factor of 4 for MVB03

→ Density dependent Michigan three range (M3Y) also predict RPA reduction of cross section (very similar results obtained for 8 parametrizations), but by a smaller factor (up to 40%)

Main results

• In the vector (spin $S = 0$) channel, one observes the expected instability at low density related to the formation of an inhomogeneous phase (=crust). At high density, there appears a zero sound mode, which does not affect appreciably the cross section.

• In the axial (spin $S = 1$) channel, one may have a spin instability ($\sim$ onset of ferromagnetism) or a spin zero sound mode at high density (3-7 times saturation density), depending on which interaction is used

• Phenomenological effective interactions (Skyrme, MSB) show instability at high density in spin sector

• Microscopical (BHF) do not show such a feature
• M3Y interactions with can represent a viable alternative

• Caveat for model APR98+CP03: Parametrization available up to $n_B = 0.24\text{fm}^{-3}$ only. Role of transition observed by APR98 around $\sim 2n_{\text{sat}}$, interpreted as pion condensation? Role of tensor force?

• We need information on the behavior of the spin Landau parameters G_0, G_0' ($\leftrightarrow$ spin asymmetry energies) as a function of density ρ and asymmetry parameter $\alpha = (\rho_n - \rho_p)/\rho$

$\boxed{5}$ INFLUENCE OF CHEMICAL COMPOSITION
e.g. Role of hyperons

▷ Some references for this section:

[1] S. Reddy, M. Prakash and J.M. Lattimer, Phys. Rev. **D58** (1998) 013009

[2] L. Mornas, Eur. Phys. J. **A23** (2005) 365, *ibid.* **A24** (2005) 293

Models of the hyperonic interaction

Here we show the result of a nonrelativistic calculation. We need the expression of the *polarized* energy density functional with hyperons. Two types of interactions were considered:

- Skyrme interaction *à la* Lanskoy *et al.*
  [PRC55 (97) 2330, PRC58 (98) 3351]:
  npe$\mu\Lambda$ matter with $\Lambda - N$ and (slightly attractive) $\Lambda - \Lambda$ interaction
- Extension to Σ^- hyperon with help of results by Dabrowski from Nijmegen potential [Acta Phys. Pol. **36** (2005) 3063]
  with attractive (model D) or repulsive (model F) $N\Sigma^-$ interaction potential. The latter case is favored by experimental data

As before, one useses relations between chemical potentials (with non-vanishing μ_ν for trapped neutrinos) and charge conservation to obtain the chemical composition of matter

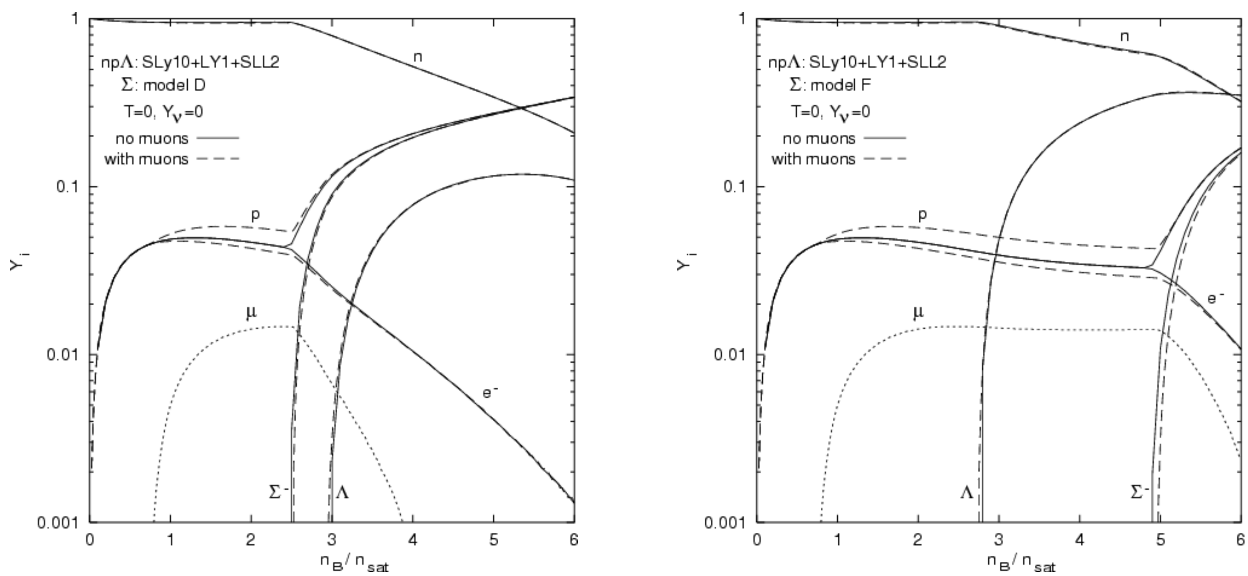


Fig. 10 – Chemical composition of $np\Lambda\Sigma^-e^-\mu^-$ matter in β equilibrium.

Evolution of the hyperonic content in a deleptonizing proto neutron star

We take the results from Fig. 17 of Pons, Reddy, Prakash, Lattimer & Miralles [Astrophys. J. **513** (1999) 780] as representative of the evolution the temperature and leptonic content.

t [s]	0	5	10	15	20	25	30	35	40	45	50
T [MeV]	17.3	25.0	31.7	36.5	37.5	36.5	33.6	29.8	26.0	22.1	18.3
Y_L	0.345	0.315	0.283	0.256	0.239	0.222	0.202	0.178	0.145	0.119	0.09

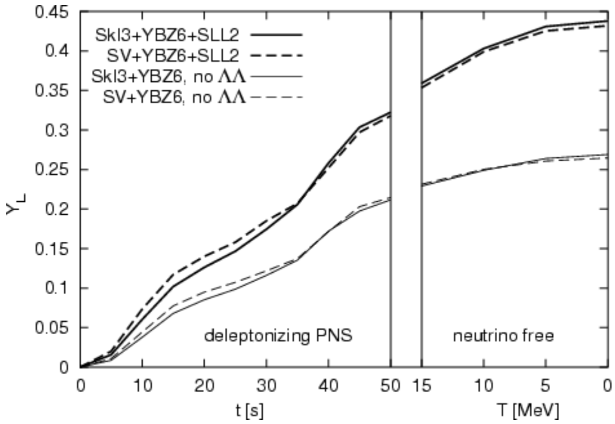
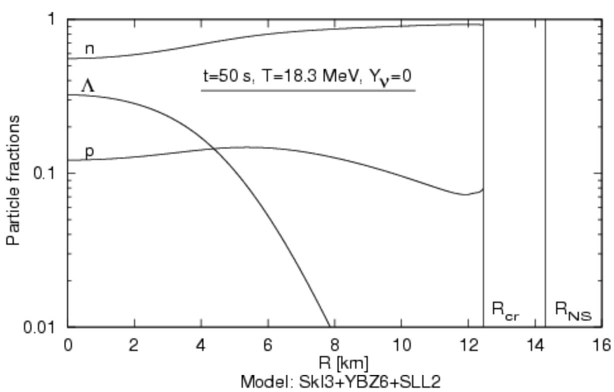
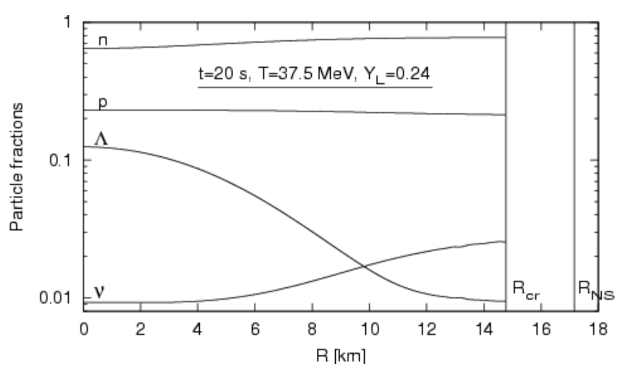


Fig. 11 – Evolution of the hyperonic content as a function of time





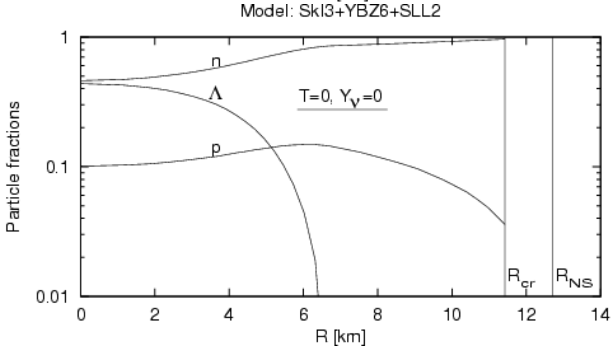


Fig. 12 – Particle fractions as a function of the neutron star radius at three stages of the deleptonization.

Influence on onset of spin instability

The ferromagnetic criterion is now given by the determinant of the 4x4 matrix

$$Det[\delta_{ij} + G_{ij}] = Det \begin{pmatrix} (1+G_0^{nn}) & G_0^{np} & G_0^{n\Lambda} & G_0^{n\Sigma^-} \\ G_0^{pn} & (1+G_0^{pp}) & G_0^{p\Lambda} & G_0^{p\Sigma^-} \\ G_0^{\Lambda n} & G_0^{\Lambda p} & (1+G_0^{\Lambda\Lambda}) & G_0^{\Lambda\Sigma^-} \\ G_0^{n\Sigma^-} & G_0^{p\Sigma^-} & G_0^{\Lambda\Sigma^-} & (1+G_0^{\Sigma^-\Sigma^-}) \end{pmatrix}$$

• As we have seen earlier, Skyrme interactions always (save one exception = old SV force) give rise to a ferromagnetic instability in npe matter. However, if the threshold for hyperon formation is lower than that for the onset of ferromagnetism in npe matter, then the ferromagnetic transition can be avoided

• Forces yielding smaller $\rho^H_(thr)$ and softer e.o.s. also tend to be more efficient in lifting the ferromagnetic criterion above the critical zero axis.

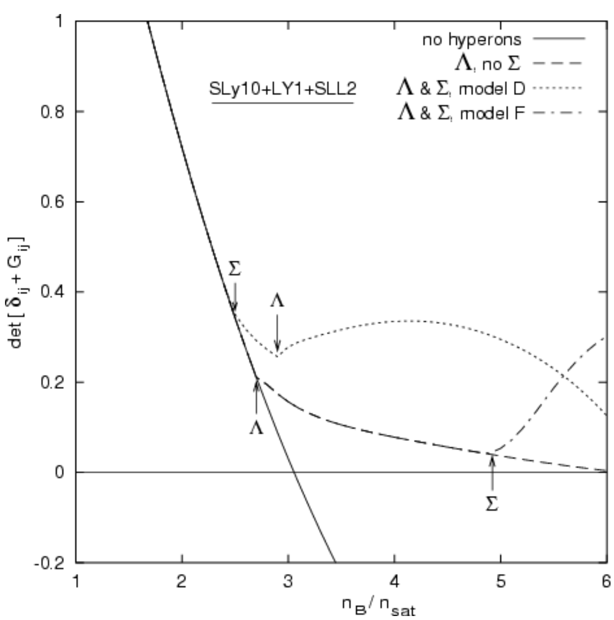


Fig. 13 – Ferromagnetic criterion in $np\Lambda\Sigma^-e^-\mu^-$ matter in β equilibrium.

Mean free path in the presence of hyperons

▷ In the mean field approximation

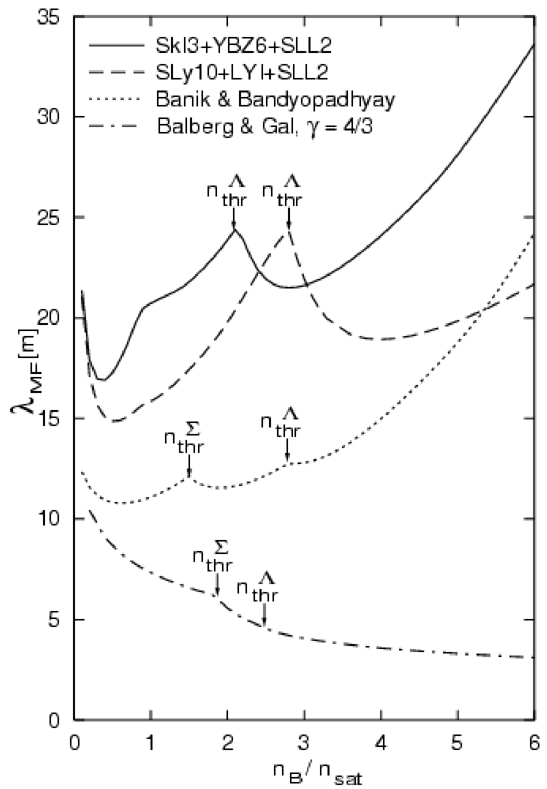


Fig. 14 – Mean free path in the mean field approximation at $T = 10$ MeV, $E_\nu = 3\,T$, $Y_\nu = 0$ for various nonrelativistic models of baryonic matter

▷ RPA corrections (Here calculated with Λ hyperons only)

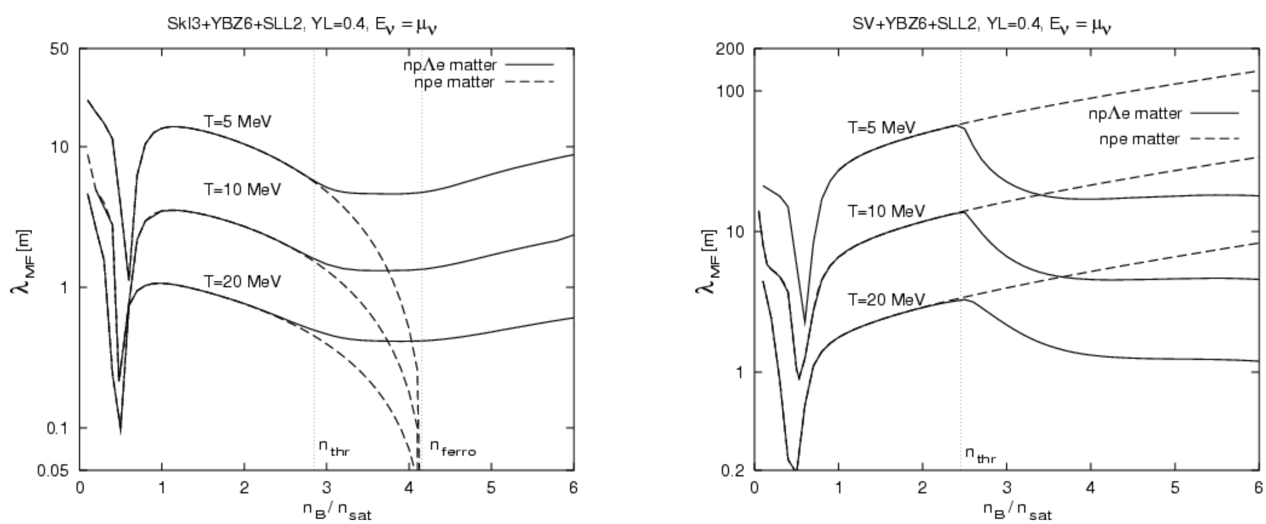


Fig. 15 – Mean free path λ_{RPA} with and without hyperons in the random phase approximation in protoneutron star matter with trapped neutrinos ($Y_L = 0.4$). Left/right = with/without spin instability in *npe* matter

$\boxed{6}$ SUMMARY

→ Protoneutron stars characterized by finite temperature and trapped neutrino content. The neutrinos diffuse out in $\sim$ 20-50 s

→ In dense matter, neutrino scattering with nucleons is modified by nuclear correlations

→ It is generally found that the neutrino opacities are suppressed by medium effects. If, however, a collective mode is excited, there is a sizeable enhancement. This can be studied in the Random Phase Approximation.

→ A mechanical instability appears in response to density fluctuations in the vector channel at subnuclear density, corresponding to clustering in the crust (nuclei + fluid of dripped neutrons)

→ In relativistic models with σ, ω, ρ, δ and π exchange (without Fock contributions), the total neutrino-neutron scattering cross section is found to be reduced at high density by RPA correlations by a factor 10% to 25% with respect to the mean field result. When the residual contact interaction g' is treated covariantly, varying its strength does not affect the diffusion (=neutral process).

→ For nonrelativistic models, one may have a spin instability ($\sim$ onset of ferromagnetism) or a spin zero sound mode at high density (3-7 times saturation density), depending on which interaction is used

• Phenomenological effective interactions (Skyrme, MSB) show instability at high density in spin sector

• Parametrizations of microscopical calculations (BHF) do not show such a feature

• M3Y interactions with can represent a viable alternative

→ An appreciable fraction of hyperons can be present in he last phases of proto neutron star cooling ($\sim$ between 30 and 50 s after collapse)

→ When no ferromagnetic transition is present, the mean free path is reduced by the presence of hyperons

→ In there is a spin instability in *npe* matter, and if the threshold for hyperon formation is lower than that for the onset of ferromagnetism in npe matter, then the ferromagnetic transition can be avoided

▷ Possible future studies

→ Wanted: information on the behavior of the spin asymmetry energies as a function of density and isospin asymmetry

→ Role of tensor force?

→ Neutrino diffusion in inhomogeneous matter