Electromagnetic Pulsar Spindown

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Abstract

We review the recent developments on the issue of electromagnetic pulsar spindown of a misaligned rotator. We evaluate them based on our experience from two idealized cases, that of an aligned dipolar rotator, and that of a misaligned split-monopole rotator, and thus obtain a different spindown expression.

We next incorporate the effect of magnetospheric particle acceleration gaps. We argue that near the death line aligned rotators spin down much slower than orthogonal ones. We test this hypothesis through a Monte Carlo fit of the P-Pdot diagram without invoking magnetic field decay. We predict that the older pulsar population has preferentially smaller magnetic inclination angles and braking index values $n>3$. Finally, we offer an observational test of our hypothesis.

The oblique pulsar magnetosphere: Spitkovsky 2006

"Modeling of the structure of the highly magnetized magnetospheres of neutron stars requires solving for the self-consistent behavior of plasma in strong fields, where field energy can dominate the energy in the plasma. This is difficult to do with the standard numerical methods for MHD which are forced to evolve plasma inertial terms even when they are small compared to the field terms. In these cases it is possible to reformulate the problem and instead of solving for the plasma dynamics in strong fields, solve for the dynamics of fields in the presence of conducting plasma. This is the approach of force-free electrodynamics (FFE)."

Spitkovsky 2006 applied the finite-difference time-domain (FDTD) numerical method commonly used in electrical engineering to solve the FFE equations that describe the evolution of an initially dipolar magnetosphere which begins to rotate obliquely. The strength of the method is very low numerical dissipation, which is useful in the study of the current sheets and consequent strong discontinuities that develop in the magnetosphere.

The oblique pulsar magnetosphere: Spitkovsky 2006

An important question is what is the electromagnetic luminosity of the inclined rotator, since currently the vacuum formula

$$L_{vac} = \frac{B_*^2 r_*^6 \Omega_*^4}{6c^3} \sin^2 \theta$$

is commonly used to infer the magnetic field of pulsars.

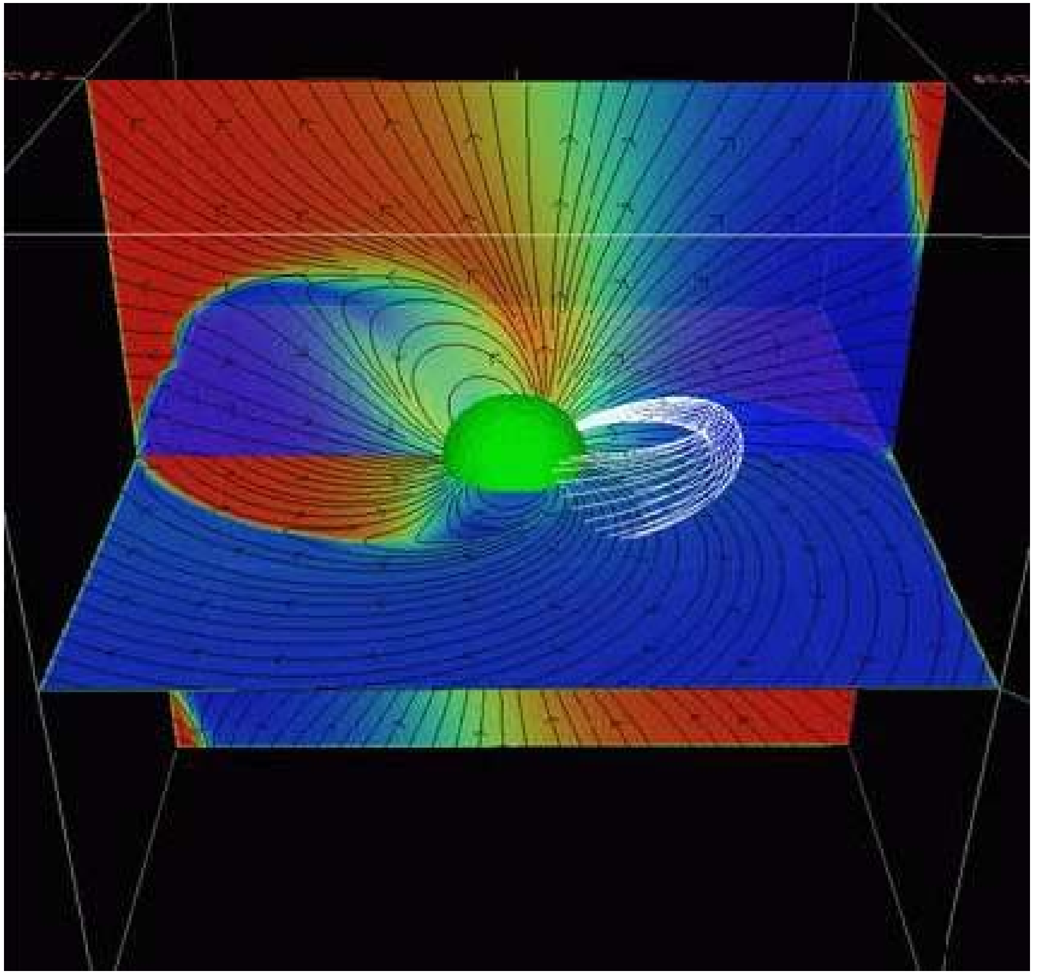
Spitkovsky 2006 evolved an oblique dipolar magnetosphere for about 1.2 turns of the star (fig. 1). He claims that the solution very quickly settles to a constant electromagnetic energy flux which depends on the magnetic inclination (fig. 2), and he obtains the formula

$$L_{pulsar} = \frac{B_*^2 r_*^6 \Omega_*^4}{4c^3} (1 + \sin^2 \theta)$$

We will now try to reproduce this result from basic principles. We will start with a short presentation of the aligned rotator case.

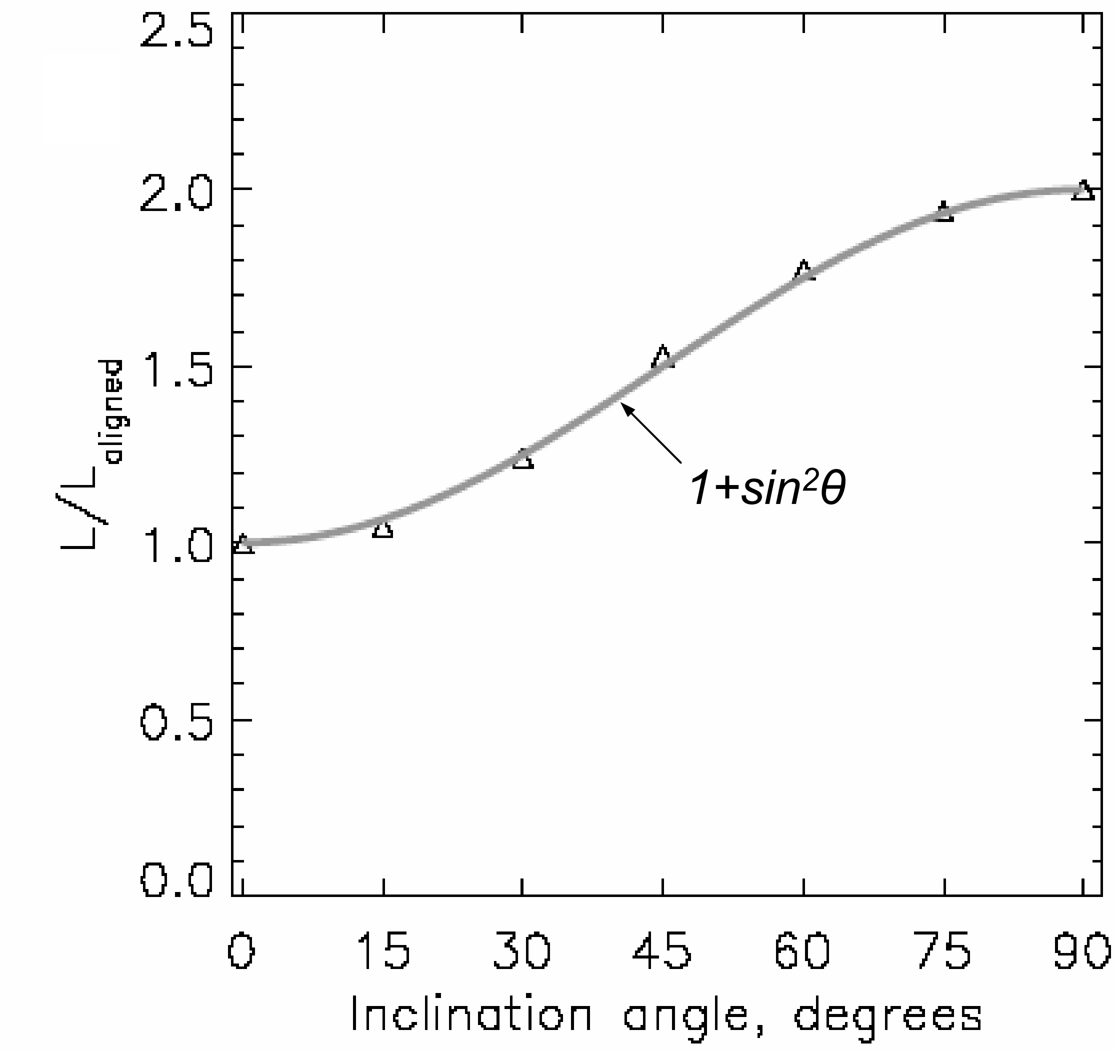
The oblique pulsar magnetosphere: Spitkovsky 2006

Fig.1: Slices through the 60° magnetosphere. Shown are fieldlines in the horizontal and vertical plane, color on the vertical place is the perpendicular field, on the horizontal plane the toroidal field. A sample 3D flux tube is traced in white.



The oblique pulsar magnetosphere: Spitkovsky 2006

Fig.2: Spindown luminosity in units of the aligned force-free luminosity (see text below) as a function of inclination. Triangles represent simulation data.



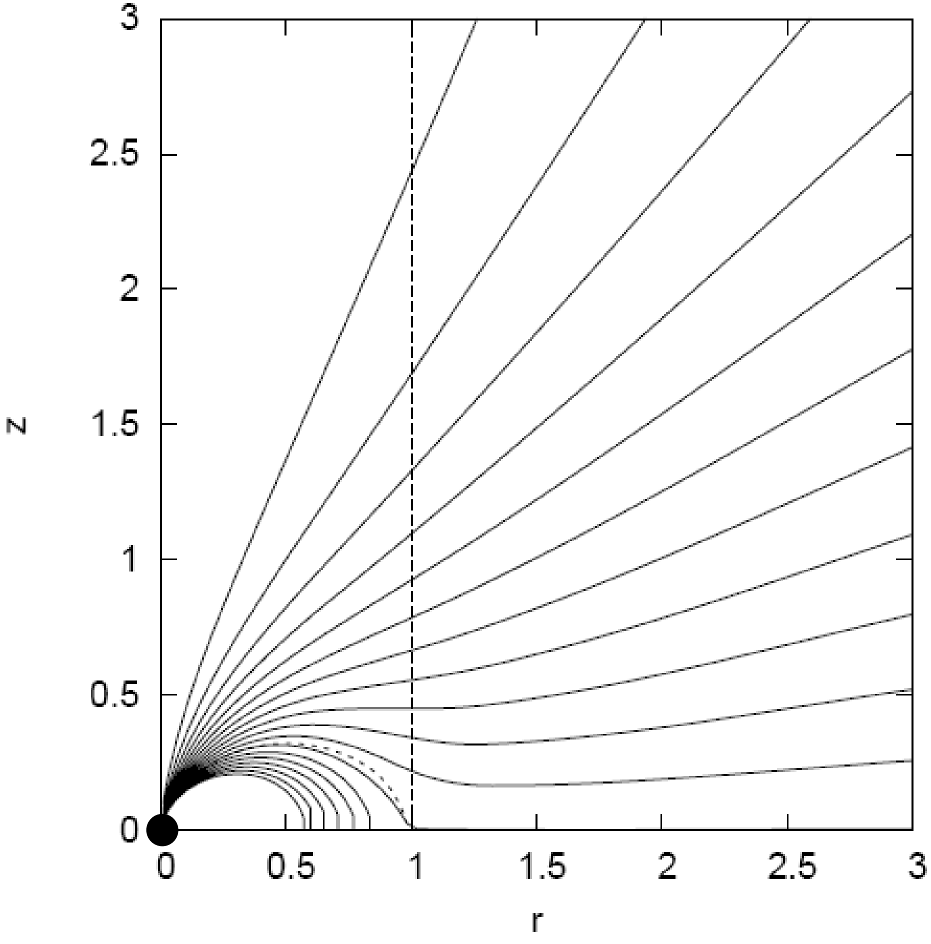
The aligned pulsar magnetosphere: CKF

Contopoulos, Kazanas & Fendt (1999, hereafter CKF) first obtained the structure of the axisymmetric pulsar magnetosphere with a dipolar stellar magnetic field (fig. 3). We showed that the asymptotic structure is that of a magnetic split monopole: a certain amount of initially dipolar magnetic flux Ψ_{open} stretches out radialy to infinity in one hemisphere, and returns to the star in the other hemisphere, forming a current sheet discontinuity along the equator. Electromagnetic spindown is due to the establishment of a poloidal electric current distribution $I=I(\Psi)$ which flows to infinity along open field lines and returns to the star through an equatorial current sheet that joins at the light cylinder with the separatrix between open and closed field lines. CKF obtained Ψ_{open} and the distribution $I(\Psi)$ self consistently by requiring that the solution be continuous and smooth at the light cylinder.

The CKF solution has since been confirmed, improved and generalized in Gruzinov 2005a, Contopoulos 2005, Timokhin 2005, Komissarov 2005, McKinney 2005 and Spitkovsky 2006.

The aligned pulsar magnetosphere: CKF

Fig.3: The axisymmetric pulsar magnetosphere (CKF). Thin lines correspond to Ψ intervals of $0.1\Psi_{dipole}$. $\Psi=0$ along the axis. The dotted line shows the separatrix $\Psi_{open}=1.23\Psi_{dipole}$. Distances are scaled to the light cylinder distance $r_{lc}=c/\Omega_*$. A poloidal electric current flows along the open field lines and returns along the equator and the separatrix between open and closed field lines. No current flows in the closed line region.



The aligned pulsar magnetosphere: CKF

The amount of initially dipolar field lines that stretches out radialy to infinity is obtained numerically as

$$\Psi_{openCKF} = 1.23\Psi_{dipole} \approx \left(\frac{3}{2}\right)^{1/2} \Psi_{dipole}$$

where

$$\Psi_{dipole} \equiv \frac{B_* \Omega_* r_*^3}{2c}$$

is defined in the magnetostatic dipole problem as the amount of dipolar flux that crosses a cylinder with radius equal to the light cylinder radius. Interestingly enough, in the magnetostatic dipole problem where field lines are required to open up beyond a distance equal to the light cylinder radius (i.e. with a current sheet discontinuity along the equator) the amount of open flux is found to be also closely equal to the above value Ψ_{open}!

The aligned pulsar magnetosphere: CKF

The distribution *I(Ψ)* is found to be very close to that of the Michel 1991 split-monopole solution (fig. 4)

$$I(\Psi) \sim I_{Michel}(\Psi) \equiv \Psi\left(2 - \frac{\Psi}{\Psi_{open}}\right)$$

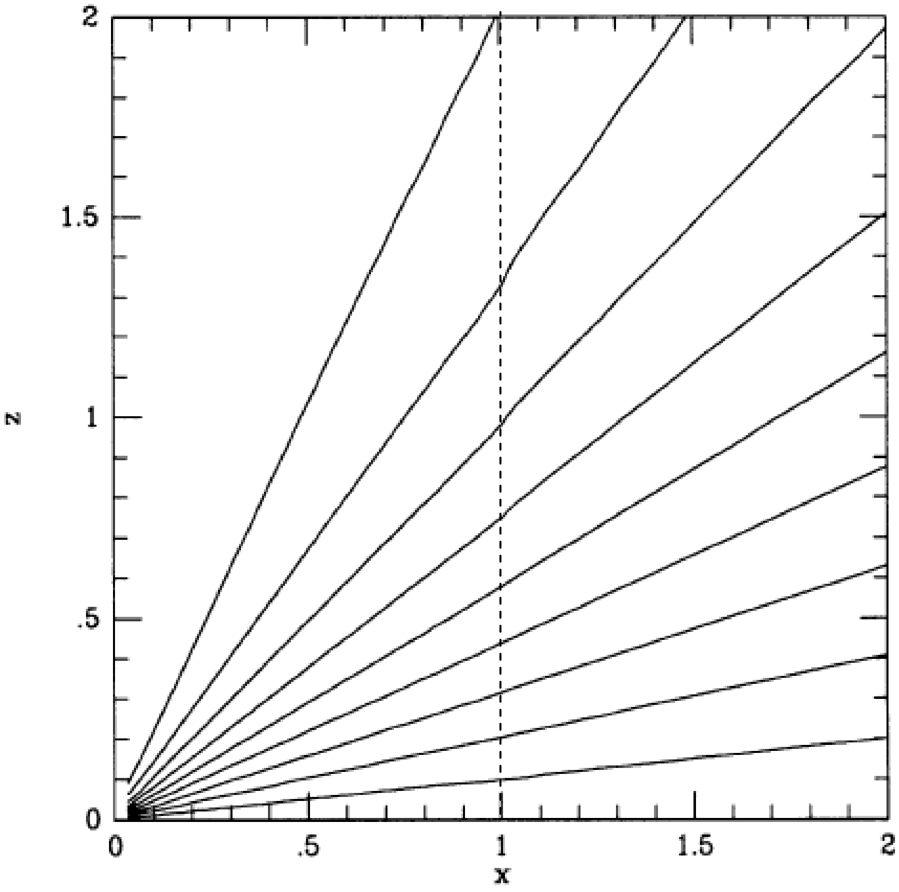
The electromagnetic spindown luminosity is thus obtained as

$$\boxed{L_{aligned} \equiv \frac{\Omega_*^2}{c}\int_{\Psi=0}^{\Psi_{open}} I(\Psi)\,d\Psi \approx \frac{2}{3}\frac{\Omega_*^2}{c}\Psi_{open}^2 \approx \frac{B_*^2 r_*^6 \Omega_*^4}{4c^3}}$$

Note that, in the axisymmetric case, the spindown luminosity may be obtained if one calculates the amount Ψ_{open} of open field lines, since the magnetosphere approaches asymptotically the Michel split-monopole solution which depends only on Ψ_{open}. As we will see next, this result is also valid in the general case of an oblique rotator.

The aligned split-monopole: Michel 1991

Fig.4: The axisymmetric split-monopole magnetosphere (Michel 1991). Thin lines correspond to Ψ intervals of $0.1\Psi_{open}$. $\Psi=0$ along the axis. Distances are scaled to the light cylinder distance $r_{lc}=c/\Omega_*$. A poloidal electric current flows along the open field lines and returns along the equator. In this idealized case, there is no closed line region.



The oblique split-monopole: Bogovalov 1999

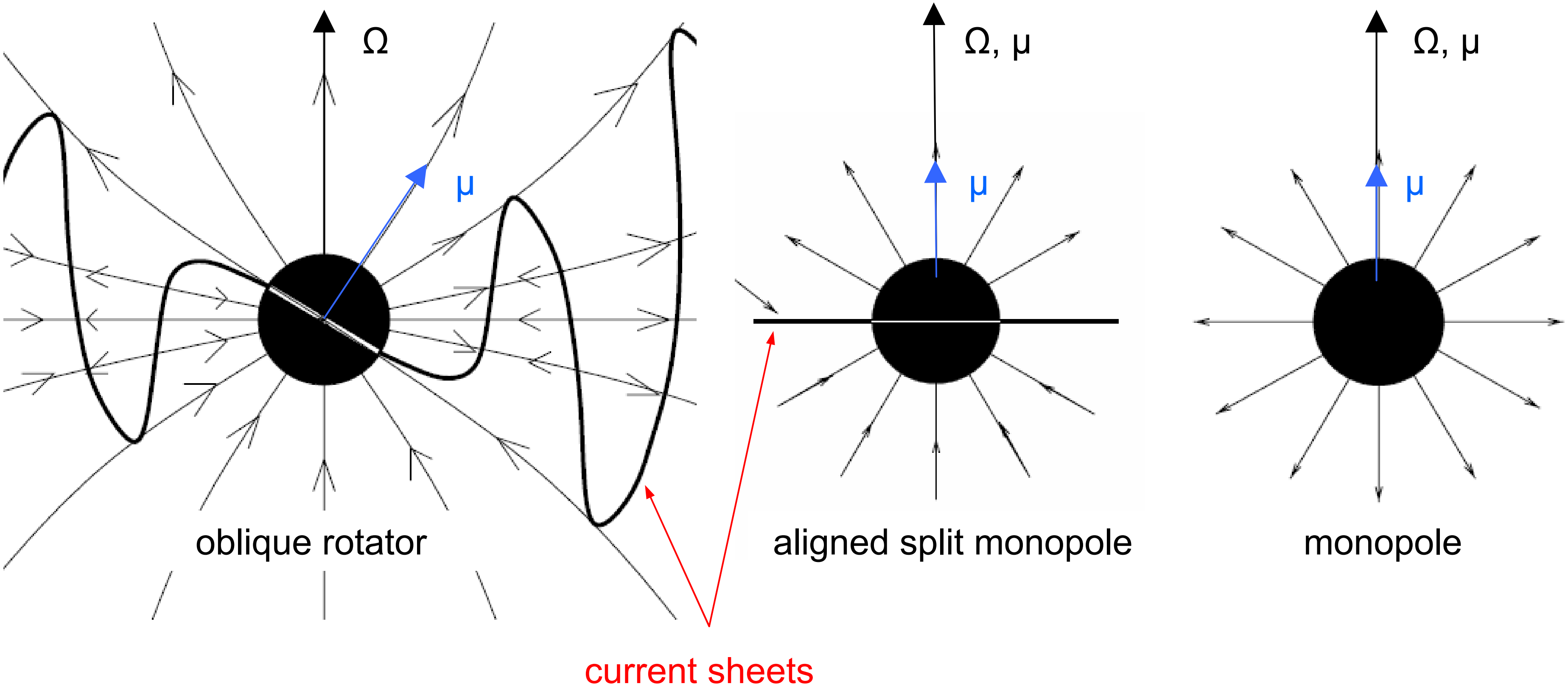
When studying the oblique rotator, it is natural to start from the simplest case, that of an oblique split-monopole. Bogovalov 1991 showed that this problem is reduced to the problem of the axisymmetric split-monopole rotator (fig.5). All properties of the cold MHD plasma flows obtained for the axisymmetric rotator are the same for the oblique rotator as well. In other words: as long as current sheet discontinuities are present to guarantee magnetic flux conservation, the direction of the magnetic field does not matter.

In particular, rotational losses of the oblique split-monopole rotator do not depend on the inclination angle, but depend only on the amount Ψ_{open} of open field lines.

We may now derive an important conclusion about the oblique dipole rotator which too, as we argued, becomes asymptotically split-monopole-like: rotational losses of the oblique rotator depend indirectly on the inclination angle through the amount of open field lines Ψ_{open} .

The oblique split-monopole: Bogovalov 1999

Fig.5: The solution in the model with the oblique split-monopole magnetic field may be obtained from the solution for the monopole magnetic field through the introduction of current sheets and the appropriate changes of sign of the magnetic field.



The oblique dipole rotator: crude estimate

We have now come full circle and are in a position to estimate the electromagnetic spindown luminosity as a function of the inclination angle through an estimate of Ψ_{open} . We may obtain a crude estimate of Ψ_{open} by calculating the amount of magnetic flux that crosses the light cylinder and originates from an inclined magnetostatic dipole at the origin, and rescale it by the CKF factor $(3/2)^{1/2}$ (see fig. 6)

$$\Psi_{open} \sim 1.23 \frac{B_* \Omega r_*^3}{2c} (0.4 + 0.6 \cos\theta)^{1/2}$$

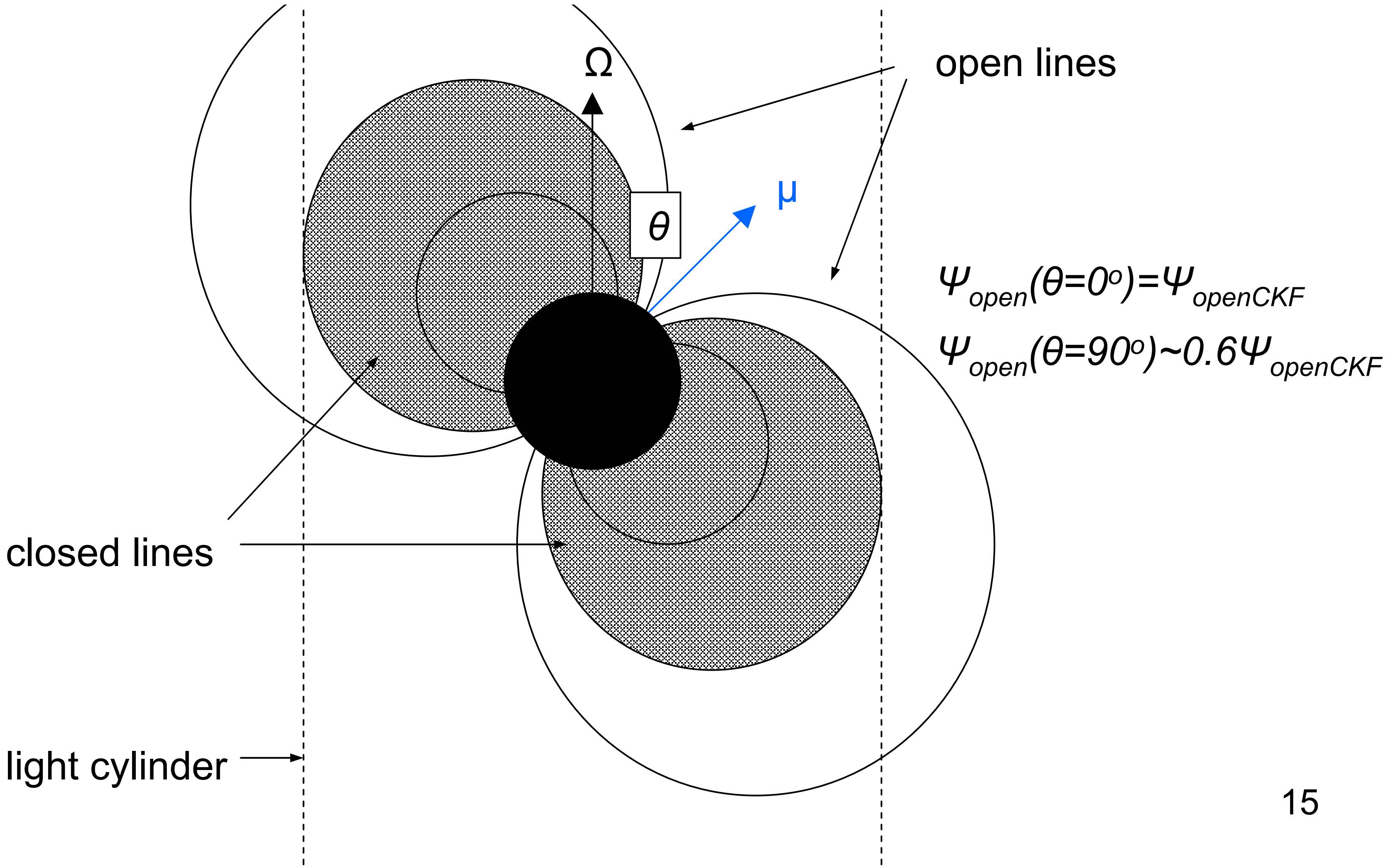
This yields the following crude estimate of the spindown luminosity

$$L_{pulsar} \sim \frac{B_*^2 \Omega_*^4 r_*^6}{4c^3} (0.4 + 0.6 \cos\theta)$$

In particular, we see that $L_{pulsar}(\theta=90^o) < L_{pulsar}(\theta=0^o)$, contrary to what Gruzinov 2005b and Spikovsky 2006 obtain. This discrepancy will need further analysis in order to be resolved.

The oblique dipole rotator: crude estimate

Fig.6: The geometry used for a crude estimate of Ψ_{open}. Note that $\Psi_{open}(\theta=90^o)$ is smaller than $\Psi_{open}(\theta=0^o)$, hence we expect that $L_{pulsar}(\theta=90^o) < L_{pulsar}(\theta=0^o)$, contrary to what recent numerical simulations imply.



Magnetospheric gaps: Contopoulos & Spitkovsky 2006

The force-free MHD picture that we have been trying to develop implies that charged particles are freely available in the magnetosphere and support the required magnetospheric space charges and electric currents. Those particles are produced in magnetospheric potential gaps where a certain amount equal to about V_{gap}~10^{13}*Volt* of the total electromotive potential which develops along the surface of the rotating star, $V_* \sim \Omega_* \Psi_{open}/c$, is "consumed" in order to accelerate them. The effective remaining magnetospheric potential

$$V = \frac{(\Omega_* - \Omega_{death})\Psi_{open}}{c}$$

is used to establish the force-free MHD solution. Here, $\Omega_{death}=V_{gap}c/\Psi_{open}$. Obviously, when Ω_* drops below Ω_{death}, electric currents cannot be supported in the magnetosphere, pulsar emission ceases, and the neutron star spins down as a vacuum oblique rotator.

Magnetospheric gaps: Contopoulos & Spitkovsky 2006

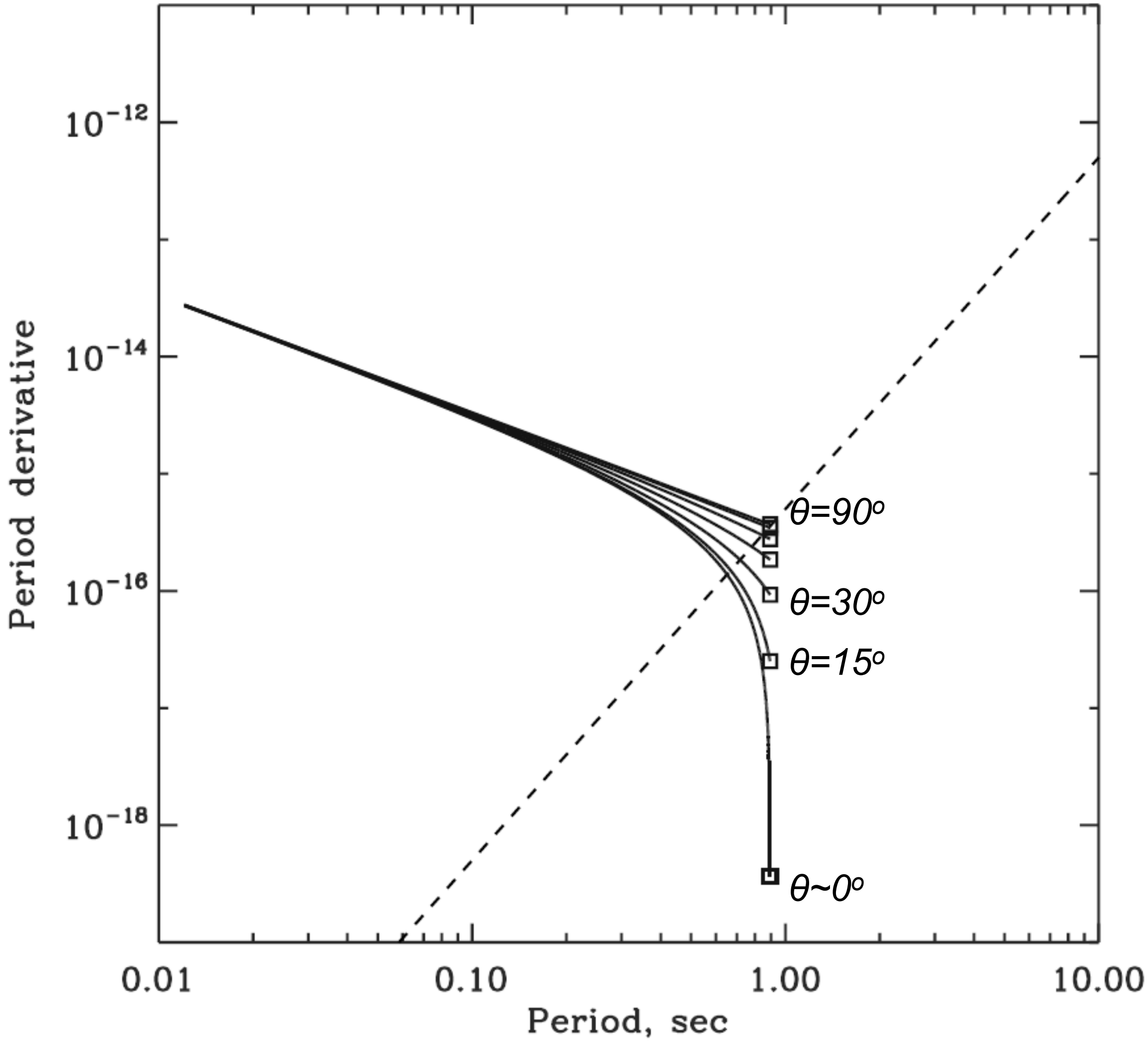
We obtained an estimate of the spindown luminosity in the presence of magnetospheric particle acceleration gaps, namely

$$L_{pulsar} \sim \frac{B_*^2 r_*^6 \Omega_*^4}{4c^3}\left(\underbrace{\alpha \sin^2\theta}_{\text{oblique rotator contribution}} + \underbrace{\left[1 - \frac{\Omega_{death}}{\Omega_*}\right]}_{\text{magnetospheric gap contribution}} \underbrace{\cos^2\theta}_{\text{aligned rotator contribution}} \right)$$

We applied this expression with *α=1* (our previous result suggests that *α* may be equal to *0.4*) to study how pulsars spin down (fig. 7), and were able to reproduce the observed distribution of pulsars in the P-Pdot diagram with the minimum number of assumptions possible, through a simple Monte Carlo numerical experiment (figs. 8-10).

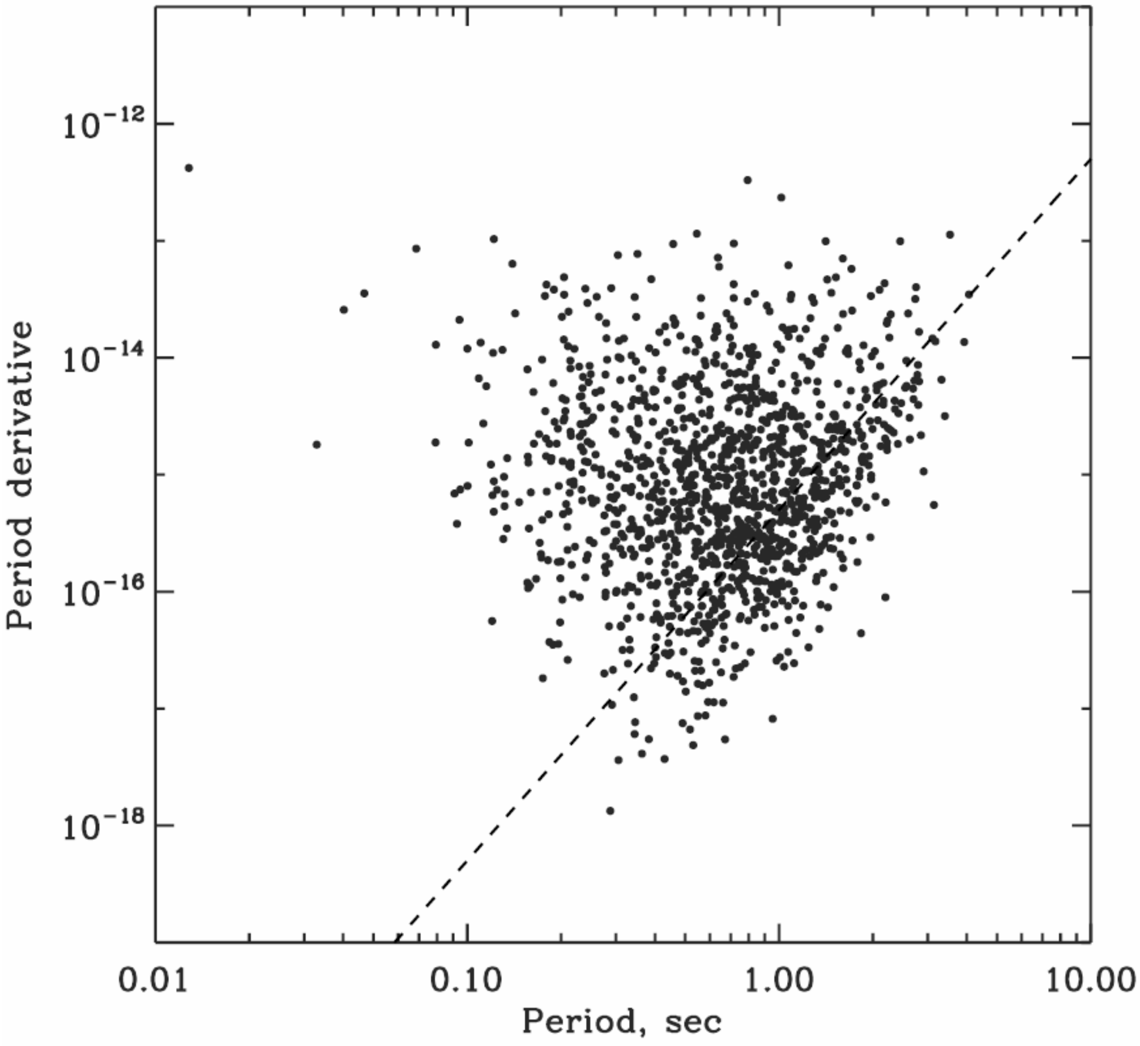
Magnetospheric gaps: Contopoulos & Spitkovsky 2006

Fig.7: P-Pdot evolutionary diagram that shows the effect of the misalignment angle θ. Rectangular dots indicate when pulsars reach Ω_{death} and turn off. The dashed line represents the theoretical death line which does not take into account the misalignment angle dependence. Oblique pulsars evolve faster through the diagram.



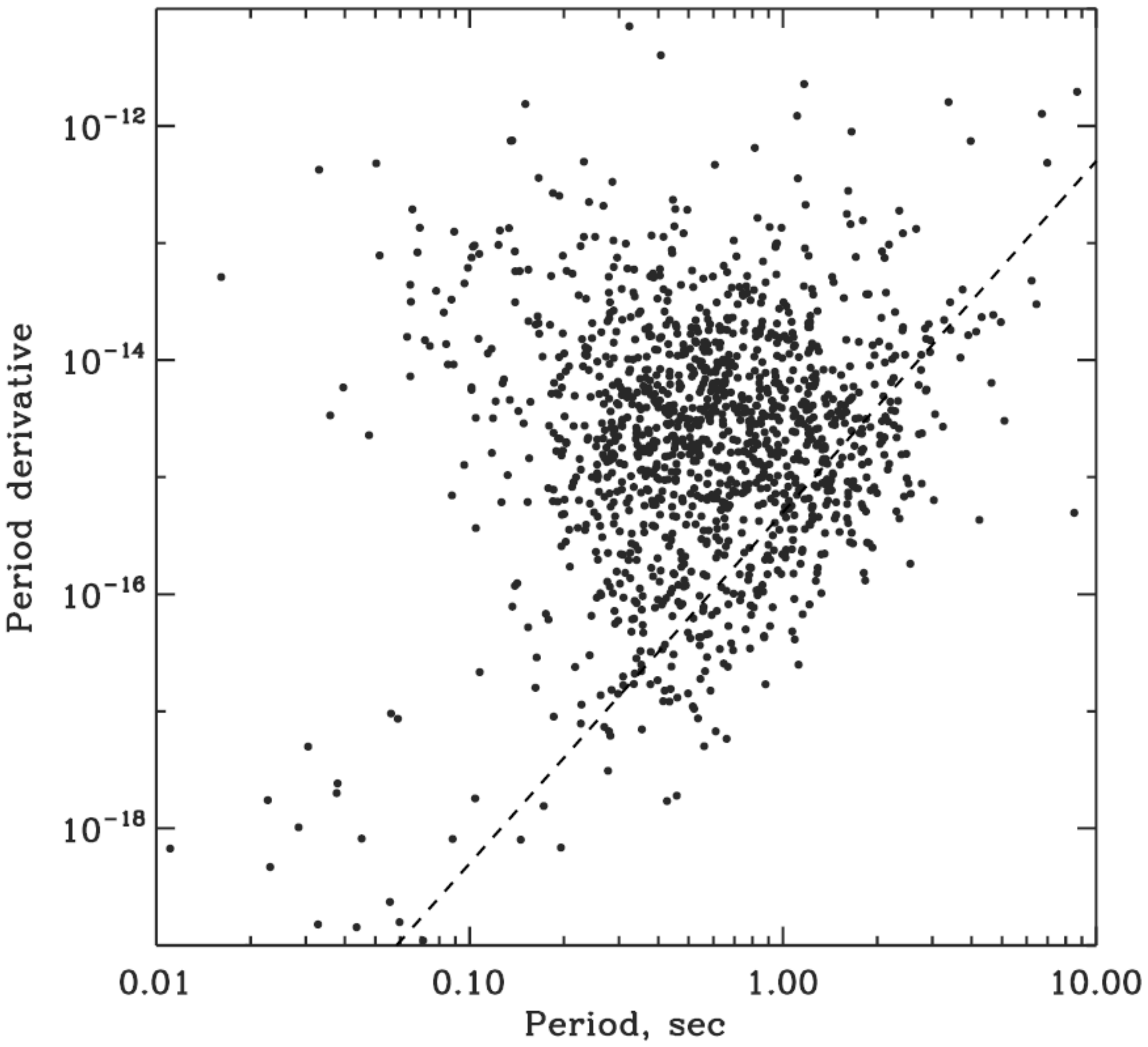
Magnetospheric gaps: Contopoulos & Spitkovsky 2006

Fig.8: Monte Carlo experiment using our new electromagnetic spindown expression. Pulsars are injected with a distribution of polar magnetic field values around $B_{*}=10^{12}G$, and initial periods at pulsar birth uniformly distributed between *10 msec* and *0.2 sec*.



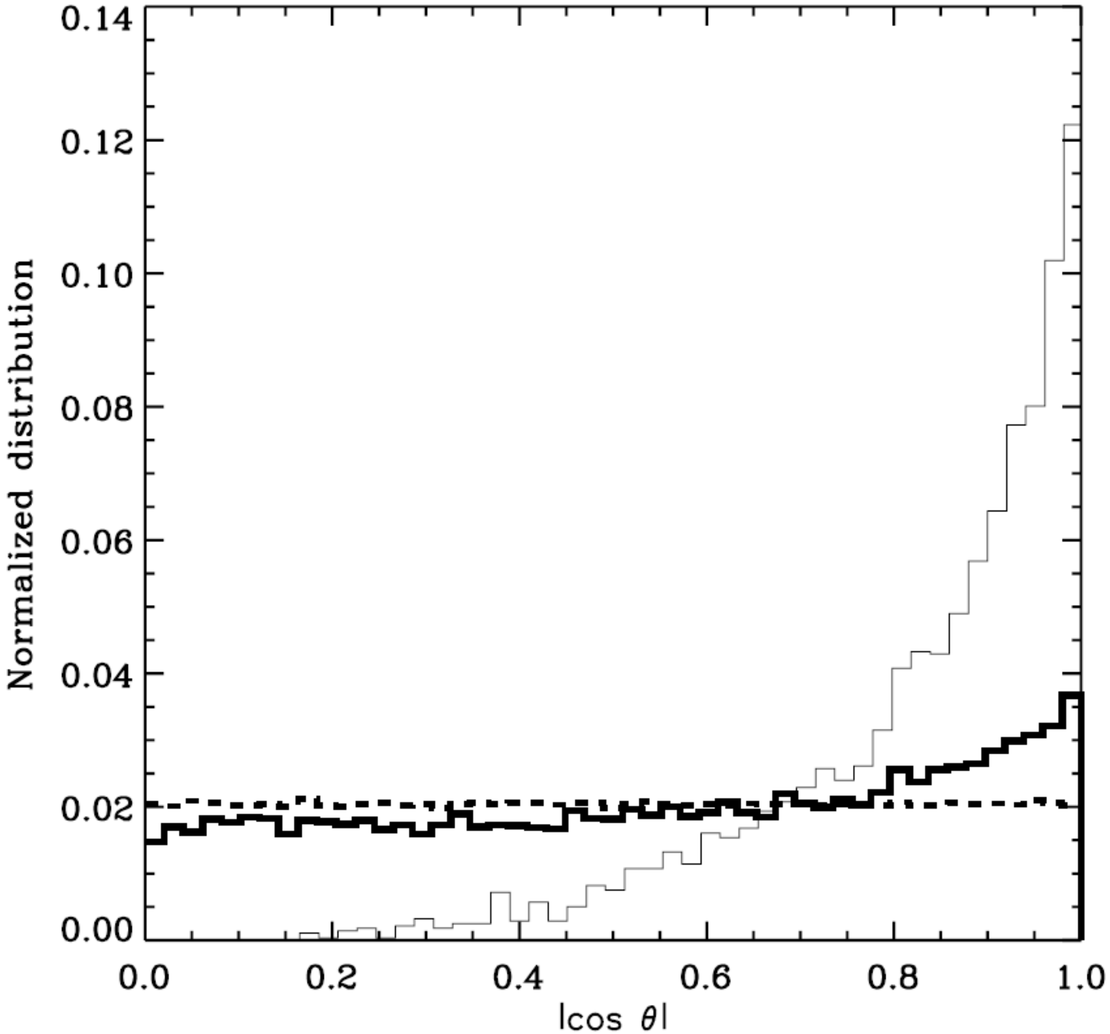
Magnetospheric gaps: Contopoulos & Spitkovsky 2006

Fig.9: The present day observed radio pulsar distribution for comparison with the result of our Monte Carlo experiment. Data from ATNF pulsar catalog.



Magnetospheric gaps: Contopoulos & Spitkovsky 2006

Fig.10: The distribution of |*cosθ*| in our Monte Carlo experiment. The dashed line represents the original distribution injected with uniform random misalignment angle. The thick solid line corresponds to the simulated pulsar distribution as a whole. The thin solid line corresponds to pulsars observed lying below the theoretical death line.



Magnetospheric gaps: Contopoulos & Spitkovsky 2006

Our main conclusions are:

1. The energy loss close to the pulsar death is smaller than what is given by the standard dipolar spindown formula. This effect gives a good fit between the theoretical and observed distributions of pulsars near the death line, without invoking magnetic field decay.
2. The model may account for individual pulsars spinning down with braking index $n<3$. However, $n\sim 3$ remains a good approximation for the pulsar population as a whole.
3. Pulsars near the death line have braking index values $n>3$. Such high braking index values may be observable.
4. Pulsars near the death line may have preferentially smaller inclination angles.

A preliminary look at the ATNF pulsar catalog data suggests that the last point may have some observational support (fig. 11).

Magnetospheric gaps: Contopoulos & Spitkovsky 2006

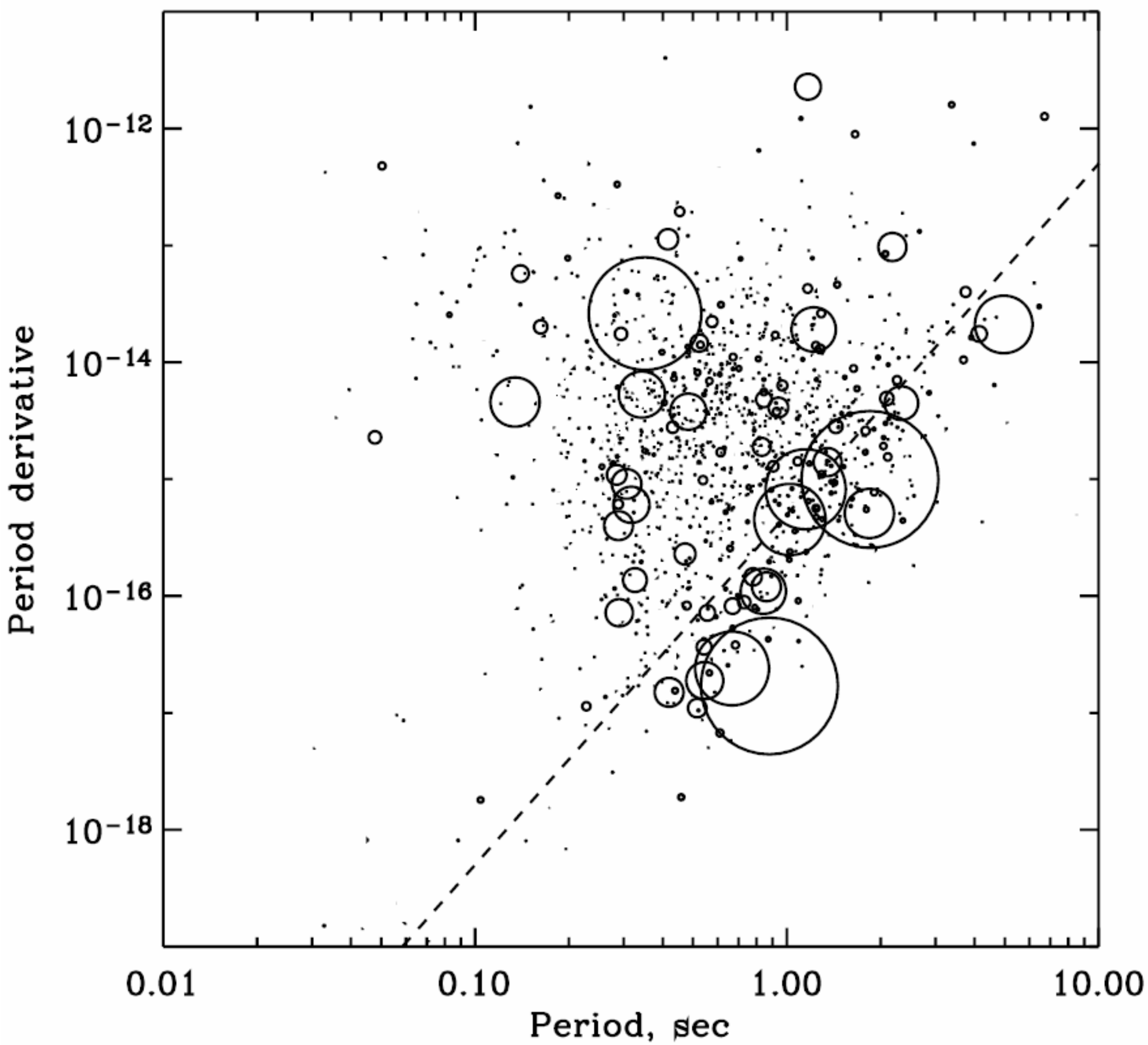
One possible measure of the inclination angle of a pulsar is its fractional pulse width, or the ratio of the width of the pulse to the period of the pulsar. If the radio beam size is independent of inclination, then we would expect pulsars with smaller inclinations to be seen for a larger fraction of the period than pulsars with large inclinations.

In order to test this hypothesis we took the available data for the pulse width at 50% of the pulsar peak from the ATNF catalog (1375 pulsars with $W_{50\%}\neq 0$). In order to be able to compare the fractional pulse width for pulsars of different periods, we have to correct for the intrinsic size of the pulsar beam. Therefore, the quantity that we relate to the degree of alignment is $F_{align}\equiv WP^{-1/2}$, where W is the measured pulse width. We plot the observed pulsars as circles with the radius of the circle linearly proportional to F_{align}.

A visual inspection of the plot shows that there is an excess of larger pulse fractions for older pulsars, and in particular for pulsars beyond the pulsar death line. This feature is particularly interesting and may serve as an indication of alignment.

Magnetospheric gaps: Contopoulos & Spitkovsky 2006

Fig.11: Distribution of alignment measure in the observed pulsar population. Each pulsar is plotted as a circle with radius proportional to the quantity $WP^{-1/2}$ using the pulse width W at 50% of the peak (see text). Data from ATNF pulsar catalogue.



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