Magnetised Electron-Positron Plasmas

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Electrostatic oscillations in cold electron-positron plasmas can be coupled to a propagating electromagnetic mode if the background magnetic field is inhomogeneous. Previous work considered this coupling in the linear regime, successfully simulating the electromagnetic mode. Here we present a stability analysis of the non-linear problem, perturbed from dynamical equilibrium, in order to gain some insight into the modes present in the system. Preliminary results from the non-linear numerical simulations are also presented.

Why Study Pulsar Pair Plasmas?

Pulsars emit radiation over a large frequency range from radio to gamma ray emission.

Pulsar spectra display complicated structure produced by radiation mechanisms in atmosphere.

Main Radiation mechanisms ignore collective effects and concentrate on single particle approach.

Single particle approach can't produce TeV photons.

Stochastic radiation recoil causes density fluctuation in pulsar atmosphere.

Pair plasma responds with plasma density oscillation

Cold plasma oscillation parallel to magnetic field is unstable to density growth.

Density instability can be quenched by radiating energy from the oscillation.

Cold plasma oscillations in inhomogeneous magnetic field couple to electromagnetic waves.

Radiative damping of oscillations acts as co-moving seat of coherent radiation in observers frame.

Full non-linear description will reveal rich structure in emission spectra as a result of collective plasma effects.

Electromagnetic Wave Coupling

Coupling of electrostatic oscillation to electromagnetic waves possible if the background magnetic field is inhomogeneous. Casting problem in 2-D using cylindrical coordinates with an axial magnetic field and an electric field in the radial and azimuthal directions, governing cold plasma equations are:

$$r\partial_t n_\pm + (r n_\pm u_{\pm r})' = 0$$
$$\partial_t u_{\pm r} + u_{\pm r}u'_{\pm r} - u_{\pm\theta}^2/r = \pm(e/m)(E_r - u_{\pm r}B_z)$$
$$\partial_t u_{\pm\theta} + u_{\pm r}u'_{\pm\theta} + u_{\pm r}u_{\pm\theta}/r = \pm(e/m)(E_\theta - u_{\pm r}B_z)$$
$$(rE_r)' = (e/\varepsilon_0)r(n_+ - n_-)$$
$$(rE_\theta)' = -r\partial_t B_z$$
$$B'_z = -\partial_t E_\theta/c^2 - \mu_0 e(n_+u_{+\theta} - n_-u_{-\theta})$$
$$0 = -\partial_t E_r/c^2 - \mu_0 e(n_+u_{+r} - n_-u_{-r})$$

There are now two plasma modes:

1. Electromagnetic wave $\omega^2 > 2\omega_p^2 + \omega_c^2$

Radially propagating with an azimuthal electric field and an axial magnetic field.

2. Electrostatic oscillation $\omega^2 = 2\omega_p^2 + \omega_c^2$

The oscillation is a radial expansion plus torsional twist.

Dynamical Equilibrium

In the equilibrium situation $\partial_t \equiv 0$, $n_- = n_+$ and $E_r = E_\theta = 0$. The resulting dynamical equilibrium initial condition is described by,

$$u_{\pm r} = \sqrt{\kappa_1^2 - u_{\pm\theta}^2} \qquad n_\pm = \frac{\kappa_0}{r\sqrt{\kappa_1^2 - u_{\pm\theta}^2}}$$

$$u'_{\pm\theta} = -\frac{u_{\pm\theta}}{r} - \frac{eB_z}{m} \qquad B'_z = -2e\mu_0\kappa_0\frac{u_{\pm\theta}}{r\sqrt{\kappa_1^2 - u_{\pm\theta}^2}}$$

Where $u_{r+} = u_{r-}$, $u_{+\theta} = -u_{-\theta}$ and κ_0, κ_1 are constants

Stability Analysis

Prior to numerically solving the governing equations need to check the stability of the system.

Linearise set of equations and look at large values of r.

In this regime approximate the equilibrium azimuthal velocity to be zero, $u_{\pm\theta} = 0$.

This requires the equilibrium magnetic field to be a constant, B_0 , and the equilibrium radial velocity to be a constant, $u_{\pm r} = u_0$, to be consistent with dynamical equilibrium.

1. Electrostatic Solution

The electrostatic oscillation is characterised by $B_z = B_0$, $E_\theta = 0$ and $u_{+\theta} = u_{-\theta}$. Substituting these conditions into the governing equations yields,

$$\ddot{\delta} + 2u_0\dot{\delta}' + u_0^2\delta'' + u_0\frac{\dot{\delta}}{r} + u_0^2\frac{\delta'}{r} + (\omega_c^2 + 2\omega_p^2)\delta + C_1 = 0$$

where $\delta = u_{-r} - u_{+r}$ and C_1 is a constant. This has solution,

$$\delta(r,t) = f_1(t - r/u_0)J_0(\nu) + f_2(t - r/u_0)Y_0(\nu) + C_1/(\omega_c^2 + 2\omega_p^2)$$

Where $\nu = \sqrt{\omega_c^2 + 2\omega_p^2}\,r/u_0$ and $f_{1,2}$ are functions. This solution describes the decay of the radial velocity of the plasma as it is convected.

2. Convective Solution

If $\dot{B}_z = \delta = 0$ and $E_\theta \sim 1/r \neq E(t)$ this yields,

$$\dot{\Delta} = \dot{\sigma} = 0 \qquad \Sigma = -2eE_r/\omega_c m \qquad \Sigma = u_{+\theta} + u_{-\theta}$$

$$\dot{E}_r = -\mu_0 c^2 e u_0 \gamma \qquad r\dot{E}_r + u_0(rE_r)' = 0 \qquad \Delta = u_{+\theta} - u_{-\theta}$$

$$\sigma = u_{-r} + u_{+r}$$

where $\gamma = n_+ - n_-$. This has solution,

$$E_r(r,t) = \frac{f(r - u_0 t)}{r}$$

Here the radial electric field is being convected at the streaming velocity of the plasma, u_0 .

3. General Solution

If prescribe $E_\theta = f(\zeta)/r$ where $\zeta = kr - (\omega \pm ku_0)t \equiv kr - \Omega t$, this implies $B_z = \beta f(\zeta)/r$, $\sigma = \lambda f(\zeta)/r$ and

$$\Delta = \frac{\Omega(1 - \beta^2c^2)}{\mu_0 c^2 ner}\frac{df(\zeta)}{d\zeta} + \frac{c^2\beta}{\mu_0 c^2 ner^2}f(\zeta)$$

where β , λ are constants and the function f is defined by,

$$\frac{\Omega(1 - c^2\beta^2)(u_0 k - \Omega)}{\mu_0 c^2 ne}f_{\zeta\zeta} + \frac{\beta(u_0 k - \Omega)}{\mu_0 ner}f_\zeta + \left[\frac{2eu_0\beta}{m} + \omega_c\lambda - \frac{2e}{m} - \frac{u_0\beta}{\mu_0 ner^2}\right]f = 0$$

The solution of which is a Bessel function of non-integer order. Solution shows presence of Doppler effect as EM wave propagates in a moving medium relative to observer.

Numerical Simulations

Solve self consistent set of non-linear equations using a Finite Difference Time Domain method (FDTD).

Solution of equation governing the dynamical equilibrium of the system gives the initial conditions of the problem.

Providing a cold electron-positron plasma has an inhomogeneous magnetic field, any electrostatic perturbation will be able to generate an electromagnetic wave.

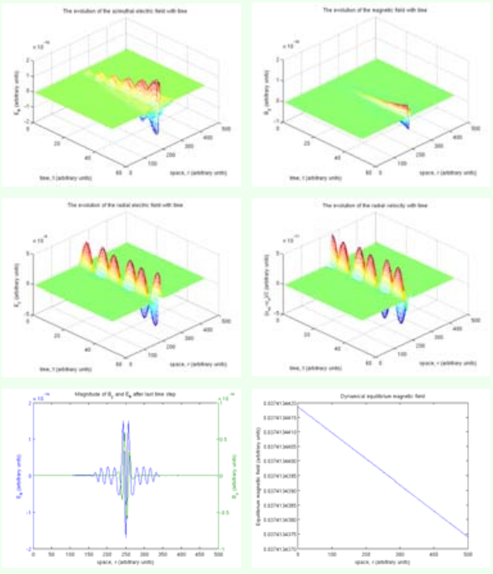
The non-uniform magnetic field induces an azimuthal drift current producing a fluctuating axial magnetic field, this couples to the azimuthal electric field.

Seat of electromagnetic radiation in plasma rest-frame.

This mechanism will certainly play a role in quenching the density instability found in the plasma oscillation by radiating away the energy of the large amplitude oscillation before the onset the unstable density spikes.

Results

The following results were obtained from the numerical simulations.



Future Developments

Continuation of non-linear code development.

Extend treatment to kinetic pair plasma. Electrostatic oscillation propagates parallel and perpendicular to magnetic field: Bernstein modes. Possibility of electromagnetic wave coupling to Bernstein modes.

Look at calculation of kinetic modes in inhomogeneous magnetic field.

Introduction of temperature to the plasma would quench density instability apparent in cold electrostatic oscillation.

Acknowledgements. We are grateful for funding from PPARC via grant number PP/C000234/1 and a studentship for C. R. Stark.